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ABSTRACT

The effects of students' grade level (four, five, or six) and of five variables related to types of open arithmetical sentences on students' ability to solve open-sentence problems were investigated. The five variables were: (1) operation symbol, (2) sentence type as determined by the symmetric property of the equality relation, (3) the position of the placeholder, (4) whether the solution was a whole number or not, and (5) whether the largest number in the problem occurs as a product in a basic fact. Two instruments incorporating these variables were developed and administered to 1298 students in grades four through six. Responses were coded and analyzed using the Fortap Statistical Package. Data from students whose performance meets predetermined minimal students were further analyzed using ANOVA and Wilcoxen signed rank tests. These analyses indicated that sentences with whole number solutions were significantly easier than others. Variables (1), (2), and (3) were also related to significant differences in subject performance, and several significant interaction effects were found. (SD)

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Technical Report No. 375
(Part 1 of 2 Parts)

DIFFERENTIAL PERFORMANCE OF FOURTH- THROUGH SIXTH-GRADE STUDENTS IN SOLVING OPEN MULTIPLICATION AND DIVISION SENTENCES

by

Mary Jane McMaster

Report from the Project on
Conditions of School Learning and Instructional Strategies

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Principal Investigator

Wisconsin Research and Development
Center for Cognitive Learning
The University of Wisconsin
Madison, Wisconsin

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ABSTRACT

Differential Performance of Fourth- Through Sixth-Grade Students in Solving Open Multiplication and Division Sentences

Mary Jane McMaster

Under the Supervision of Professor J. Fred Weaver

The Problem

The purpose of this study was to find out whether differences exist in pupils' performance when solving selected types of open multiplication and division sentences derived from the form a o b = c.

Procedure

Specifically, this investigation sought to find out the differences in students' responses to open number sentences when the following factors were varied: (A) school grade (4, 5, and 6), (B) the symbol for the operation specified in a sentence (x or ÷), (C) sentence type as determined by the symmetric property of the equality relation (a o b = c versus c = a o b), (D) the position of the placeholder in a sentence (a, b, or c), (E) the existence or non-existence of an open sentence solution within the set of whole numbers (■ x b = 20 versus ■ x 5 = 21), and (F) the largest number being a basic fact product or not a basic fact product in open sentences which have no whole number solution (3 x ■ = 25 versus 3 x ■ = 23).

Two distinct kinds of multiplication and division open sentence tests were constructed and administered to 1298 fourth-, fifth-, and sixth-grade students from eight schools. Each student was administered a 28-item open sentence number puzzle test (NPT) and a 14-item basic multiplication and division test (BMDT).

The data furnished by all 1298 subjects were corrected and coded by the investigator. The information was then key punched for computer analysis and analyzed by the Fortap Statistical Package. This process yielded descriptive statistical results. The data furnished by students who responded correctly to at least four of the five nontrivial multiplication items and four of the five nontrivial items on the BMDT, were further analyzed by ANOVA and Wilcoxen Signed Ranks Test.

Results

1. The performance level of subjects on open sentences having whole number solutions was significantly different between grade levels.
2. The performance level of subjects on open multiplication sentences was significantly different from the performance level of subjects on open division sentences.
3. The performance level of subjects on operation-left open sentences was significantly different from the performance level of subjects on operation-right open sentences.

4. The performance level of subjects on open sentences was significantly different for placeholder positions a, b, and c.
5. Significant interactions existed among the following factors: grade level, operation, symmetric property, and placeholder position.
6. The performance level of subjects on open number sentences which have no whole number solutions was significantly different from the performance level of subjects on open sentences which have whole number solutions.
7. Relative to the open sentences with no whole number solutions, there was no significant difference between students' performance level on open sentences in which the largest number was a basic fact product, and students' performance level on open sentences in which the largest number was not a basic fact product.

Conclusion

The analysis was complex to interpret because of the significant interactions. There appeared to be a very high interaction between operation division and placeholder position a. Significant interactions also existed between the following factors: (1) grade and operation; (2) grade and symmetric factor; (3) grade and placeholder position; (4) operation and symmetric factor; (5) operation, symmetric factor, and grade; (6) operation and placeholder position; (7) symmetric factor and placeholder position; and (8) operation, symmetric factor, and placeholder position. Caution must be exercised, therefore, in taking an overly simplistic interpretation of significant differences between levels of principal factors.

Nevertheless, the evidence warrants the belief that much greater attention needs to be given to principal factors B, C, D, and E in preparation of text materials and in instruction pertaining to open multiplication and division sentences.

Chapter I

INTRODUCTION

The purpose of this study was to find out whether differences exist in pupils' performance when solving selected types of open multiplication and division sentences derived from the form $a\ o\ b = c$.

Specifically, this investigation sought to find out the differences between fourth-, fifth-, and sixth-grade students' responses to open sentences when the following mathematical factors were varied: (a) the operation specified in a sentence [$\times$ and $\div$], (b) sentence type as determined by the symmetric property of the equality relation [$a\ o\ b = c$ versus $c = a\ o\ b$], (c) the position of the placeholder in a sentence [$\blacksquare\ o\ b = c$, $a\ o\ \blacksquare = c$, or $a\ o\ b = \blacksquare$], (d) the existence or nonexistence of an open sentence solution within the set of whole numbers, e.g., $\blacksquare \times 5 = 20$ versus $\blacksquare \times 5 = 21$, and (e) the largest number in the sentence being a basic fact product or not a basic fact product in open sentences which have no whole number solution, e.g., $3 \times \blacksquare = 25$ versus $3 \times \blacksquare = 23$.

Previous Research

Weaver (1971) conducted a study which investigated student responses to open number sentences. The investigation involved first-, second-, and third-grade students. The two operations investigated were addition and subtraction. Effects of the symmetric property were examined. Weaver studied the effect of the placeholder in each of the three positions (■ o b = c, a o ■ = c, and a o b = ■). A part of the study also investigated the students' ability to recognize open sentences which had no whole number solution. Weaver's study revealed that differences in students' responses to open number sentences do exist.

Grouws (1971) also investigated open number sentences. Among other things, Grouws investigated the relative difficulty of four open sentence types involving addition and subtraction. His study sought to reaffirm that differences do exist in students' responses to open number sentences, and also, by means of an interview technique, to investigate how the students thought about the different open sentence types.

Importance of Open Sentences

Grouws (1971) discussed at length the mathematical and pedagogical significance of open sentences. The following six statements summarize Grouws' discussion of the mathematical importance of open sentences.

1. The symbolic nature of open sentences makes them useful as an aid in formulating clear and precise statements of mathematical relationships.

2. Collections of open sentences are frequently used to define algebraic structures.

3. The special class of open sentences called equations have been studied by scholars since antiquity. The result of this study is the area of mathematics called the theory of equations.

4. Equations can be used in studying various algebraic structures and in the study of field theory.

5. Equations and other kinds of open sentences are an integral part of the study of many branches of mathematics.

6. Open sentences play an important role in mathematical model building [p. 2-4].

Grouws summarized his discussion of the significance of open sentences by stating that:

> open sentences are essential in formulating clear and precise statements of important mathematical and physical relationships. Open sentences are important mathematical entities, and they are valuable in the construction of mathematical models of the physical world [p. 5].

Grouws has indicated that open sentences have pedagogical as well as mathematical importance. Among the reasons Grouws discussed as the pedagogical importance of open sentences were the following:

1. Open sentences have been used in elementary school mathematics programs since the 1940's.

2. The use of open sentences in a specific pattern to assist a child in forming a generalization is a widely used instructional technique.

3. The use of pairs of open sentences in a similar fashion can be used to help children "discover" other important relationships.

4. Open sentences provide a means by which interdisciplinary approaches to many projects and activities, not usually studied in a mathematical lesson, can be initiated.

5. Open sentences also have important mathematical applications which can be significant for elementary school children.

6. Open sentences are important in aiding children to develop mathematical modeling ability.

7. Open sentences are useful in improving verbal problem solving [p. 5-8].

Heddens (1968) emphasized the importance of open sentences in pedagogy by pointing out that "it is virtually impossible to teach mathematics at any level without using mathematical sentences [p. 335]." An examination of contemporary mathematics textbooks reveals that the use of mathematical sentences is common place at all educational levels (Grouws, 1971, p. 5).

One can hardly doubt the importance of open sentences in the study of mathematics. Open sentences are found in first-grade contemporary mathematics programs and continue to be found through nearly all phases and realms of mathematics. They add both clarity and simplicity to the study of mathematics.

Purpose of the Study

The purpose of this study was to examine the performance of fourth-, fifth-, and sixth-grade students solving open sentences of the types in Table 1.1. Were the levels of performance by students at each grade level significantly different as the open sentence types were altered?

If a student can respond "eight" when asked the solution to $16 \div 2 = \blacksquare$, it has often been assumed that he knows that number fact. However, when it comes time to test students, teachers often change the open sentence type presented to the student. The reasoning seemed to be that if the students really knew the number facts, the open sentence type would be unimportant. This implies that the difficulty level of an open sentence is determined solely by the number combination involved. If this is true, then the difficulty level of an open sentence is not affected by the open

Table 1.1

Open Sentence Types Investigated

Operation		Operation Left-Symmetric	Operation Right-Symmetric
Multiplication	1	# a x b = ■	# ■ = a x b
	2	* a x ■ = c	* c = a x ■
	3	* ■ x b = c	* c = ■ x b
Division	4	* a ÷ b = ■	* ■ = a ÷ b
	5	* a ÷ ■ = c	* c = a ÷ ■
	6	# ■ ÷ b = c	# c = ■ ÷ b

* These sentence types were also used for the no whole number solution open sentences.

# These sentence types were not used for the no whole number solution open sentences since any whole number assignment to a and b in the multiplication open sentences would yield a whole number solution. Similarly, any whole number assignment to b and c in the division open sentences would yield a whole number solution.

sentence type as determined by the left-right symmetric property and the position of the placeholder within the open sentence.

Before the discussion of the problem, it is advantageous to clarify the meaning of the terms employed in that discussion. The following definitions were chosen to conform to standard usage.

Definitions of Terms

Open Sentence

A mathematical sentence in which there exists one or more variables may be called an open sentence. Once the variables are replaced by constants, the sentence is either true or false. Number sentences can be long or short. For this investigation, all the number sentences involved two whole numbers and one placeholder. In many cases, a whole number solution existed for the placeholder. In others, no whole number solution was possible.

Number Puzzles, Problems, Number Equations

The sentences investigated were open sentences involving two whole numbers and one placeholder (e.g., $2 \times ■ = 8$). Table 1.1 lists each open sentence type studied. Within the students' textbooks, these open sentences were often referred to as problems, or as number equations. Since students sometimes associate problems with work and puzzles with fun activities, for this study it was decided to refer to the open sentences as puzzles so as to be less detrimental to the students.

Symmetric Property--Operation-Left, Operation-Right

Within this investigation, reference will be made to operation-left and operation-right open sentences. Operation-left refers to open sentences in which the operation (multiplication or division) is on the left of the equality sign: $a \circ b = c$. The six open sentence types in Column One of Table 1.1 are all operation-left. Operation-right refers to open sentences in which the operation (multiplication or division) is on the right of the equality sign: $c = a \circ b$. The six open sentence types in Column Two of Table 1.1 are all operation-right.

Placeholder Positions a, b, and c

An opaque box (■) served as the placeholder in the open sentences. Placeholder position a was the position immediately to the left of the operation sign, e.g., open sentence types in Rows Three and Six of Table 1.1. Placeholder b was the position immediately to the right of the operation sign, e.g., open sentence types in Rows Two and Five of Table 1.1. Placeholder c was the position on the opposite side of the equality sign from the operation sign, e.g., open sentence types in Rows One and Four of Table 1.1.

Canonical Form

Canonical form refers to the open sentence form in which the placeholder occurs by itself as one member of the equation, e.g., open sentences types in Rows One and Four of Table 1.1. Generally the first experiences the students have with open multiplication and division sentences are in canonical form, and more particularly operation-left (see Table 1.1 Rows One and Four, Column One).

Specific Purpose of Study

This study investigated whether or not differences exist in student performance on open number sentences across three grade levels--fourth, fifth, and sixth. This investigation also sought to find out whether there existed overall differences in student performance when the operation was division or multiplication. Weaver (1971) concluded that students performance on basic addition and subtraction open sentences was not independent of the operation.

This study investigated whether or not differences exist in student performance on open number sentences which are operation-right as compared to open number sentences which are operation-left. Some investigations have been conducted concerning these factors. With regard to the left-right symmetric property in connection with open addition and subtraction sentences, Weaver (1973) stated:

> The essence of the symmetric property of the equality relation - if X = Y, then Y = X - has been viewed all too often as something that is "intuitively obvious" to children. Consequently, it often is assumed that if a child can cope satisfactorily with statements of equality

> in a particular form (X = Y), he then can cope just as well with similar statements expressed in an equivalent symmetric form (Y = X)[3].
>
> Findings from the investigation reported here would seem to raise some doubt about the validity of such an assumption. Equivalent symmetric forms of open sentences derived from a o b = c and c = a o b were not solved equally well [p. 56].

Specifically this investigation sought to find out whether differences exist in performance in solving multiplication and division open number sentences when the open sentence type as determined by the symmetric property of the equality relation was varied. Was the performance level on operation-left sentences (i.e., 2 x 4 = ■) significantly different from the performance level on operation-right sentences (i.e., ■ = 2 x 4)?

Did the position of the placeholder in an open sentence appear to make a difference in student performance? For instance, were the performance levels of students to these three open sentence types significantly different: ■ x 3 = 12, 4 x ■ = 12, and 4 x 3 = ■ ? Studies by Weaver (1971), Grouws (1971), and Suppes (1972) all indicated that the position of the placeholder did make a significant difference in students' performance level of correct responses given to open addition and subtraction sentences.

Were students sensitive to open sentences which had no whole number solutions? Did students realize that 3 x ■ = 7, for instance, had no whole number solution? Within a previous study, Weaver (1972) stated:

> It is neither sufficient nor desirable to restrict instruction with open sentences to examples for which whole-number solutions invariably exist. The inclusion of "no solution" instances will facilitate rather than inhibit pupils' ability to work comprehendingly with open addition and subtraction sentences.

> It is important to emphasize the left-to-right order of reading number sentences (open or closed). . . . analogous convictions apply in connection with the operations of multiplication and division . . . [p. 691].

In the present investigation, the set of open number sentences with no whole number solutions was divided into two parts. One-half involved open sentences in which the largest number was not a basic fact product, and the remaining one-half involved open sentences in which the largest number was a basic fact product. For example, 3 x ■ = 13 was improvised from the number combination 3 x 4 = 12 to offer an instance of an open sentence in which the largest number was not a basic fact product. The number fact 4 x 5 = 20 was utilized to derive 4 x ■ = 21. In this case 21 was a basic fact product. If the students have never seen the products (quotients) among the basic facts, then those open sentences might have received "N" responses more often than the no whole number solution open sentences that involved basic facts numbers.

Answers to questions such as the above are necessary for textbook writers, teachers, and teacher educators. If significant differences exist, then it is necessary that these groups be made aware of the differences and prepared to deal with them. Textbook writers could incorporate more or fewer experiences with the different types of open number sentences. Teachers could offer more experiences with these different open sentence types, as well as more stress and meaningful instruction when dealing with them.

Teacher educators need to make future teachers aware of the differences, various methods to approach the teaching of these open sentence types, and the difficulties students might encounter when confronted with these various open sentence types. Teacher inservice classes need to study significant differences in students responses and discuss methods and ideas that might be effectively employed to achieve the desired outcomes.

Background of the Problem--Related Research

The most exhaustive study conducted to date concerning grade level, operation, symmetric property, and placeholders was done by Weaver (1971). The investigation involved 3,268 first-, second-, and third-grade students from 23 schools in Madison, Wisconsin. The two operations investigated were addition and subtraction. Effects of the symmetric property were examined. Weaver studied the effect of the placeholder in each of the three positions (■ o b = c, a o ■ = c, and a o b = ■). A part of the study also investigated the students' ability to recognize open sentences which had no whole number solution.

Weaver distinguished 20 types of simple open addition and subtraction sentences involving whole numbers. These have been classified in Table 1.2. A 32-item inventory was established with each item being one of the 20 types of open sentences. The study was done within the set of whole numbers, and more specifically employing basic facts having sums between 10 and 18. Basic facts were described by Weaver (1971) in the following way:

Table 1.2

Examples of Twenty Open Sentence Types Employed in Weaver Study

		SYMMETRIC PROPERTY OF EQUALITY	
		Operation-left a o b = c	Operation-right c = a o b
Open Sentence Types for Which Whole Number Solutions Exist			
Operation: Addition	Position of Placeholder: a	■ + 2 = 3	3 = ■ + 2
	b	1 + ■ = 3	3 = 1 + ■
	c	1 + 2 = ■	■ = 1 + 2
Operation: Subtraction	a	■ - 1 = 3	3 = ■ - 1
	b	4 - ■ = 3	3 = 4 - ■
	c	4 - 3 = ■	■ = 4 - 3
Open Sentence Types for Which No Whole Number Solutions Exist			
Operation: Addition	Position of Placeholder: a	■ + 4 = 2	2 = ■ + 4
	b	4 + ■ = 3	3 = 4 + ■
	c	X	X
Operation: Subtraction	a	X	X
	b	4 - ■ = 6	6 = 4 - ■
	c	3 - 5 = ■	■ = 3 - 5

X indicates no open sentence is possible (i.e., any open sentence of this type would have a whole number solution).

> A basic addition fact may be viewed as a statement of the form $a + b = c$, where a, b, and c are whole numbers such that $a < 10$ and $b < 10$; thus, necessarily, $c \leq 18$. Similarly, a basic subtraction fact may be viewed as a statement of the form $a - b = c$ where a, b, and c are whole numbers such that $b < 10$ and $c < 10$; thus, necessarily, $a \leq 18$, since $a = c + b$ [p. 513].

Results of the Weaver Study

The performance level on the 32-item inventory increased from grade 1 to grade 2 to grade 3, as can be seen in Table 1.3 (Weaver, p. 516).

Table 1.3

Mean Correct Responses

on 32-Item Inventory (Weaver Study)

Grade Level	Mean
1	12.8
2	19.1
3	22.5

The performance level was higher for addition sentences than for subtraction sentences within each grade. Table 1.4 indicates the means by grade for both the addition and subtraction sentences. The performance level on the addition sentences increased from grade 1 to grade 2 to grade 3. The performance level also increased on the subtraction sentences from grade to grade.

Table 1.4

Student Means for Addition and Subtraction Sentences

(Weaver Study)

Grade Level	Mean on Addition Sentences (16 Items)	Mean on Subtraction Sentences (16 Items)
1	7.3	5.5
2	11.1	8.0
3	13.1	9.4

As can be seen from Table 1.5, the performance level on the operation-left and operation-right sentences increased from grade to grade. The performance level was consistently higher for the operation-left sentences.

Table 1.5

Student Means for Operation-Left

and Operation-Right Sentences (Weaver Study)

Grade Level	Operation-Left (16 Items)	Operation-Right (16 Items)
1	7.5	5.3
2	10.7	8.4
3	11.8	10.7

Table 1.6 indicates the mean correct responses on open sentences having placeholders in different positions. The performance level for each of the three placeholder positions increased from grade to grade. The performance level was consistently lowest for placeholder _a_. The performance level was highest for placeholder _c_.

Table 1.6

Mean Correct Responses on Open Sentences Having Placeholders in Position _a_, _b_, and _c_ (Weaver Study)

Grade Level	Position of Placeholder					
	a		b		c	
	Mean (10	Per Cent Items)	Mean (12	Per Cent Items)	Mean (10	Per Cent Items)
1	2.7	27	5.2	43	4.9	49
2	4.7	47	7.7	64	6.7	67
3	5.9	59	9.0	75	7.6	76

Table 1.7 indicates the mean correct responses on some open sentences that do and others that do not have whole-number solutions. As can be seen, there was a tendency for the performance level on whole number solution sentences to increase from grade to grade. There was a very slight tendency for the performance level on sentences having no whole-number solutions to increase from grade to grade.

Table 1.7

Mean Correct Responses on Open Sentences That Do Have Whole-Number Solutions and Others That Do Not Have Whole-Number Solutions (Weaver Study)

Grade Level	Sentences Having Solutions Within the Set of Whole Nos.		Sentences Having No Solution Within the Set of Whole Nos.	
	Mean (24 Items)	Per Cent	Mean (8 Items)	Per Cent
1	9.5	40	3.3	41
2	15.3	64	3.8	48
3	18.4	77	4.1	51

From these results Weaver (1973) made the following conjectures:

1. It is likely that performance is not independent of open-sentence form as determined by the symmetric property of the equality relation.
2. It is likely that performance also is related to one or more of the following factors:
   a. grade level (indirectly an indicator of relative amount of experience with simple open sentences):
   b. the operation used in the statement of an open sentence:
   c. the position of the placeholder in an open sentence:
   d. the existence or nonexistence of an open-sentence solution within the set of whole numbers.
3. It is likely that some interactions exist -
   a. between (1) above and some aspect(s) of (2) above, and
   b. among aspects of (2) above [p. 55].

Grouws' Research With Respect to Difficulty of Various Open Sentence Types

Grouws' (1971) investigation continued from Weaver's (1971) study. Weaver's study revealed that differences in students' responses to open number sentences do exist. Grouws' study sought not only to reaffirm that differences do exist in students' responses to open number sentences, but also, by means of an interview technique, to investigate how the students thought about the different open sentence types.

Among other things, Grouws (1971) investigated the relative difficulty of four open sentence types involving addition and subtraction. The four open sentence types studied were a + N = b, N + a = b, a - N = b, and N - a = b. Thirty-two subjects were randomly selected from a pool of 9 third-grade classes in Madison, Wisconsin. Each subject was individually given a 16-item test in an interview situation. Each interview lasted approximately 33 minutes. The 16 open sentence types on each student's test were completely crossed and balanced with respect to the three factors Grouws was investigating: open sentence type, number size, and context. Table 1.8 indicates the percent of correct responses for each open sentence type. One may conclude that the subjects in the Grouws study gave the correct response much less frequently for the sentence type N - a = b than for the other three sentence types. These results were consistent with the results from Weaver's study (1971).

Table 1.8

Percent of Correct Responses for Each Open Sentence Type Investigated

(Grouws Study)

Open Sentence Type	$N + a = b$	$a + N = b$	$N - a = b$	$a - N = b$
Per Cent	60%	65%	37%	62%

Grouws also investigated the effect of number size in solving open sentences. One half of the open sentences involved basic facts, while the second half involved two-digit combinations. The whole numbers used in the basic facts open sentences were each less than 19. The whole numbers used in the open sentences involving larger numbers were greater than 20 and less than 100. For example, $13 - N = 8$ is an open sentence involving basic facts. $63 - N = 24$ is an open sentence involving two-digit combinations. Seventy-eight percent of the subjects gave the correct responses to the open sentences involving the basic-facts combinations. Thirty-four percent of the subjects achieved the correct responses to the open sentences involving two-digit number combinations. From this one may conclude that the size of the numbers involved in the open sentence had a direct bearing on the difficulty level of the open sentence.

Grouws also investigated two contexts which he called symbolic and verbal-symbolic. A representative problem in the symbolic context was an open sentence. A problem in the verbal-symbolic context required both an open sentence and a verbal problem appropriate to that particular open sentence. Half of each students problems were in the symbolic context. The results indicated no significant difference in performance level existed when an appropriate verbal problem was present in an open sentence solving task.

Suppes' Research Involving Open Sentence Difficulty

While Suppes (1972) was not directly investigating the difficulty of various open sentence types, there were some measures of difficulty of various sentence types embedded within his 1966-68 Stanford mathematics programs in computer-assisted instruction. Students participating in this drill and practice computer-assisted program were exposed to open sentences involving all four operations, vertical and horizontal format, canonical and noncanonical format, and differing number sizes. The placeholder appeared in all three positions, a, b, and c. For example, in their earliest experiences with addition and subtraction, subjects saw such open sentences as 2 + 0 = ?, 2 + ? = 3, 4 - 3 = ?, and ? - 4 = 0. Multiplication first appeared in grades two and three in a horizontal format in both canonical and noncanonical problems and with a maximum product of nine. The maximum product was 81 for students in grades three and four. In grades five and six the maximum product was increased to 144.

Multiplication problems in a vertical format were presented in the third grade with both one-digit by one-digit and one-digit by two-digit problems. By the sixth grade the problems were more complex with multiplication of a three-digit number by a two-digit number (p. 33). Problems appeared as 2 x 3 = ?, ? x 2 = 16 and 9 x ? = 36. In the examples given, the placeholder appeared in all three positions. Also, most of the horizontal problems were operation-left.

Division was first presented to the students in a fractional form. Each open sentence had a whole number solution, The placeholder appeared in any one of the three positions. (For example, 15 / 5 = ?, ? / 3 = 7, and 18 / ? = 9.) After the students had completed problems such as $4\overline{)28}$ = ? with the response 7, they saw one of the following - 4 x 7 = ?, ? x 7 = 28, or 4 x ? = 28, and filled in the blank.

Open number sentences involving decimals were presented in operation-right form. For example, .09 = 9 / ? and 0. 35 = ? / 100 were two such open sentences.

Within grade one, Suppes found the average probability correct as a function of sentence type (see Table 1.9). Suppes ranked 39 open addition sentences of various types by difficulty level according to highest percentage of correct responses given by second-grade students. With one exception, the first 21 rankings were operation-left and in canonical form. These received correct responses from 89% to 98% of the time. 2 + 4 = ? was ranked 1 and received 98%

Table 1.9

Average Probability Correct--Grade 1

(Suppes Study)

Form		Pretest	Posttest
Canonical	$\begin{array}{r} a \\ +\ b \\ \hline \end{array}$	.95	.95
	a + b = ?	.95	.94
Noncanonical	a + ? = c	.87	.93
	? + b = c	.82	.92

correct responses while 4 + 2 = ? was ranked 21 and received correct responses 89% of the time. Among the next nine in ranking difficulty, all were operation-left, one was in canonical form, five had the placeholder in *b* position, and three had the placeholder in *a* position. Ranked 30 was 3 + ? = 10 with observed success response of 73%. Rankings 31-39 were assigned to problem type a + b = c + d with the placeholder always in position *c* or *d*. These were successfully answered from a high of 64% of the time down to 9% of the time. 2 + 7 = ? + 4 received rank 39 and was successfully answered only 9% of the time.

This same type analysis was done again in second grade with subtraction problems. Similar results followed. The first 26 out of 47 rankings were operation-left in canonical form except for two examples in which the placeholder was in the b position. Percentage correct ranged from 99% to 88%. Of the 6 problems assigned rankings 27-32, four had the placeholder in the b position and only two were in canonical form. 10 - ? = 3 was assigned rank 32 and answered correctly 70% of the time. ? - 4 = 3 was assigned rank 33 and answered correctly 44% of the time. Problems assigned ranks 33-47 were all of two forms, ? - 5 = 2 and 10 - 4 = ? - 1. 7 - 2 = ? - 5 was assigned rank 47 and answered correctly only 4% of the time.

The above student achievement patterns were consistent throughout the grades investigated. Suppes concluded:

> The variable PB (placeholder) made a significant contribution to the prediction of performance in 10 of the 14 predictions, reflecting the fact that problems in canonical form were easier than those for which some kind of transformation was necessary [p. 74].

Although there have been other investigations pertaining to multiplication and division, these have focused more exclusively on the learning or relative difficulty of basic facts (e.g., Brownell & Carper, 1943) or on algorithms associated with these operations (e.g., Van Engen & Gibb, 1956; Cromer, 1974). Such studies did not investigate factors such as placeholder position and symmetric property of equality that are of concern in the present study.

Conclusion

From a synthesis of the previous studies, the following conclusions could be formulated:

1. The majority of the reported investigations involving grade effect, the symmetric property, and placeholder position have been concerned with the operations of addition and subtraction.
2. Subjects have been primarily from grades one, two, and three.
3. Operation-left and operation-right open sentences are not of the same degree of difficulty for students.
4. The position of the placeholder within an open sentence appears to effect the subjects' performance.
5. In general, students have not had extensive experiences with open sentences which have no whole number solutions.

Most of the previous research regarding open sentences has involved the operations of addition and subtraction. It therefore seems that research involving the operations of multiplication and division is needed with respect to open sentences. These conclusions and the previous discussion provide incentive for investigating several questions.

Principal Questions Investigated in the Study

1. Are there differences in students' performance across three grade levels--fourth-, fifth-, and sixth-grade?
2. Are there differences in students' performance when the operation is multiplication as compared to division?
3. Are there differences in students' performance when the symmetric property is applied?

4. Are there differences in students' performance when the placeholder is in the a, b, or c position?

5. Are there differences in students' performance when the open sentence has a whole number solution versus the open sentence which does not have a whole number solution?

6. Are there differences in students' performance to no whole number solution open sentences in which the largest number is a basic fact product and those in which it is not a basic fact product?

Questions of Secondary Interest

7. How frequently will students indicate no whole number solution exists to open sentences which have whole number solutions?

8. How often will students erroneously employ the inverse of the operation indicated?

9. How frequently will students respond with a number quite close to the correct solution?

10. How often will students perform the specified operation "across the equality sign?" For example, how often will subjects respond 138 to the open sentence $8 \times \blacksquare = 16$?

11. How frequently will students perform the inverse operation "across the equality sign?" For example, how often will the open sentence $15 \div \blacksquare = 3$ receive 45 as the response?

Chapter II

THE STUDY

In order to obtain information to ferret out possible differences in student responses to open number sentences, tests were administered to fourth-, fifth-, and sixth-grade children. These tests consisted of multiplication and division open sentences in which the unknown value was replaced by an opaque box (■). 8 ÷ 2 = ■ and 7 x ■ = 21 are examples of a division open sentence and a multiplication open sentence respectively. To make the tests more appealing to students, these open sentences were referred to as number puzzles. The students were to indicate for each number puzzle, the whole number that was hiding under the box, or mark "N" to indicate that no such whole number existed.

Specifically, this investigation sought to find out what differences exist between fourth-, fifth-, and sixth-grade students' responses to open sentences when the following mathematical factors were varied: (a) the operation specified in a sentence [x and ÷], (b) the sentence type as determined by the symmetric property of the equality relation [a o b = c versus c = a o b], (c) the position of the placeholder in a sentence [■ o b = c, a o ■ = c, or a o b = ■], (d) the existence or nonexistence of an open sentence solution within the set of whole numbers, and (e) the largest number being a basic fact product or not a basic fact product in open sentences which have no whole number solution.

The Tests

Two distinct types of tests were designed to administer to subjects participating in this study. Each student will respond to a 28-item open sentence number puzzle test and a 14-item basic multiplication and division test. The 28-item open sentence Number Puzzle Test will be referred to as NPT. The 14-item Basic Multiplication and Division Test will be referred to as BMDT.

The Number Puzzles Test (NPT)

Based upon the three factors--the sign of the operation specified in the open sentence, the symmetric property of the equality relation, and the position of the placeholder in the sentence, the 12 generic open sentence types identified in Table 2.1 can be generated. These sentence types are the multiplication and division counterparts of the addition and subtraction open sentences employed in the Weaver (1971) study.

Table 2.1

Generic Open Sentence Types Investigated

$a \times b = \blacksquare$	$a \times \blacksquare = c$	$\blacksquare \times b = c$
$\blacksquare = a \times b$	$c = a \times \blacksquare$	$c = \blacksquare \times b$
$a \div b = \blacksquare$	$a \div \blacksquare = c$	$\blacksquare \div b = c$
$\blacksquare = a \div b$	$c = a \div \blacksquare$	$c = \blacksquare \div b$

As the existence or non-existence of a solution within set W is considered with the 12 generic types, 20 particular open sentence types are identified (see Table 2.2). It should be noticed that there were four cells in which no entry was possible. Two of these cells were generated by the generic type a x b = ■ and its symmetric form, ■ = a x b. Since for any whole numbers a and b there always exists a whole number solution, the intersection of these generic types with non-existence of a solution is empty. Similarly, ■ ÷ b = c and c = ■ ÷ b must always have whole number solutions since given c and b whole numbers, their product must necessarily exist and be a whole number.

If the numerals 2 through 9 are assigned to a and b, respectively, 64 multiplication combinations can be generated. Sixty-four division combinations can be generated by reversing the order of the digits in each multiplication combination.

Multiplication and division combinations in which values of zero and one were assigned to a and b have unique problems associated with them. Any multiplication combination in which b is assigned a value of zero will not yield a division open sentence by the preceding method. For example, 7 x 0 = 0 becomes 0 ÷ 0, which is undefined. Multiplication combinations in which a or b are assigned a value of one are probably the most familiar to children. It was decided that these combinations (multiplication combinations in which one or zero is assigned to a or b and the division open sentences derived from these multiplication combinations) would not be a part of the formal investigation but could be utilized effectively as familiarization number puzzles on the

Table 2.2

Open Sentence Types for Number Puzzles Test (NPT)

Generic Form of Open Sentence	A Solution Exists Within W?			
	Yes		No	
	Type	Test-Number on Test	Type	Test-Number on Test
1. a x b = ■	W-1	1-1 - 3 2 - 2 3 - 2 4 - 3	#	#
2. ■ = a x b	W-2	1 - 3 2 - 2 3 - 2 4 - 3	#	#
3. a x ■ = c	W-3	1 - 1 2 - 3 3 - 1 4 - 2	N-3	1 - 1 3 - 1
4. c = a x ■	W-4	1 - 1 2 - 3 3 - 1 4 - 2	N-4	1 - 1 3 - 1
5. ■ x b = c	W-5	1 - 2 2 - 1 3 - 3 4 - 1	N-5	2 - 1 4 - 1
6. c = ■ x b	W-6	1 - 2 2 - 1 3 - 3 4 - 1	N-6	2 - 1 4 - 1

*

#--no entry
W--whole number solution exists
N--no whole number solution exists
*--table continued on next page

Table 2.2 (Continued)

Open Sentence Types for Number Puzzles Test (NPT)

Generic Form of Open Sentence	A Solution Exists Within W?			
	Yes		No	
	Type	Test-Number on Test	Type	Test-Number on Test
7. a ÷ b = ■	W-7	1 - 2 2 - 2 3 - 1 4 - 3	N-7	1 - 1 3 - 1
8. ■ = a ÷ b	W-8	1 - 2 2 - 2 3 - 1 4 - 3	N-8	1 - 1 3 - 1
9. a ÷ ■ = c	W-9	1 - 2 2 - 2 3 - 2 4 - 1	N-9	2 - 1 4 - 1
10. c = a ÷ ■	W-10	1 - 2 2 - 2 3 - 2 4 - 1	N-10	2 - 1 4 - 1
11. ■ ÷ b = c	W-11	1 - 2 2 - 2 3 - 3 4 - 2	#	#
12. c = ■ ÷ b	W-12	1 - 2 2 - 2 3 - 3 4 - 2	#	#

#--no entry
W--whole number solution exists
N--no whole number solution exists

introductory cover sheet. Since this study was concerned with the basic facts, 10 was not assigned to a or b in any multiplication combination.

The doubles combinations form a small and unique subset of the set of all basic multiplication combinations. Since the doubles are not quite like the multiplication combinations in which a $\neq$ b, the investigator decided to eliminate both the eight multiplication combinations in which a = b and the corresponding eight division open sentences.

This process yields 56 multiplication facts and 56 division facts. These 112 facts produced were partitioned into four groups of 28 items. Each group of 28 items produced the number facts for one of the four forms of the Number Puzzle Test. The four forms of the NPT will be referred to as NPT-1, NPT-2, NPT-3, and NPT-4. The systematic indicated assignment of each number combination to one of the four NPT and the generic type assignment to each number combination within each NPT (i.e., 1-28), was summarized in Appendix A.

The 56-multiplication facts were each assigned to one of the four NPT in an attempt to avoid creating the opportunity for a student to utilize the solution of one open sentence to aid in solving another open sentence. The a and b number assignments that generated the multiplication open sentences for NPT-1 were also employed to generate the 14-division open sentences for NPT-3. Similarly, the multiplication facts used in NPT-2 were used for the division combinations for NPT-4, NPT-3 multiplication facts to generate the NPT-1 division facts, and NPT-4 multiplication facts to generate NPT-2 division facts. This assignment procedure yielded the four NPT, each composed of 14-multiplication and 14-division open sentences.

In order to generate some open sentences with no whole number solutions, two-multiplication and two-division open sentences were selected from each of the four NPT. These open sentences were each altered so that no correct whole number solution was possible. The product (or dividend in division open sentences) in each open sentence was either increased or decreased by one. Within each NPT, one multiplication and one division open sentence were altered such that the product and the dividend were numbers corresponding to some basic fact. Similarly, the remaining two multiplication and division open sentences with no whole number solutions were altered such that the "product" and "dividend" were not numbers found among the basic facts.

For example, NPT-1 had the following two multiplication facts assigned to the "no whole number solution" category: 6 x 7 = 42 and 7 x 3 = 21. The first was changed to 6 x 7 = 43 (not a basic fact number--NBF) and the second to 7 x 3 = 20 (BF). Twenty is a product of the basic fact 4 x 5, while 43 is not the product of any basic fact. This distinction was made to help find out whether subjects cue on the basis of never having seen the number among the basic facts, therefore, there must be no whole number solution.

Each number combination was rewritten to correspond to the open sentence type to which the combination was assigned. These open sentences were then randomly assigned to positions 1-28 within each of their respective NPT. The resulting four NPT are indicated in Appendix B.

Basic Multiplication and Division Test (BMDT)

Questionable information would have been gained when open sentence types were varied if the students did not know the basic facts. For this reason it was desirable to find out whether or not the students originally could produce the correct response when asked some multiplication and division facts. Each student was given a Basic Multiplication and Division Test (BMDT). In order to obtain this measure and yet keep this part of the test short, each BMDT was composed of five randomly selected multiplication facts and five randomly selected division facts taken from the next higher NPT (i.e., BMDT-1 number facts were taken from NPT-2). The number facts for BMDT-4 were taken from NPT-1. These 10 number facts along with four additional number facts involving the number 'one' comprised the 14 item BMDT each student worked. The number facts involving 'one' were chosen to offer some success experiences and to make the BMDT appear to be somewhat different from the NPT.

The 14 number facts for each BMDT were then randomly assigned to positions 1-14 on each of their respective BMDT. The resulting number combinations assigned to each BMDT are indicated in Appendix C.

Since there were two parts to each student's test (NPT and BMDT), the risk of learning was present. For example, the first half of the test might have possibly affected student responses to the second half of the test. To balance for this effect, half the tests were printed with the BMDT first followed by the NPT, while the remaining half of the tests were printed in the reverse sequence. This did not eliminate any possible effects of the first part of the test on the second part, but it did at least balance them. There were four forms of the NPT and two sequences for each form. This determined eight distinct test sequences.

If the students had in their possession both halves of the test simultaneously, information from one part could have been utilized to answer open sentences on the remaining part. To avoid this problem, each student was to raise his hand after he had completed the first part of the test. At this time the test administrator or teacher collected the first half and gave that student the appropriate second half. The eight test sequences were color coded to facilitate test administration.

If the students responded correctly to the BMDT, one might suspect incorrect responses on the NPT were due to changes in the open sentence types. If the students did not reach a satisfactory performance level on the BMDT, however, it would have been difficult to determine whether incorrect responses on the NPT were attributable to lack of knowledge of the basic facts or to the variations in open sentence types. For this reason a cut-off point was established for part of the analysis of the data. Summary and descriptive analyses were employed on all the data. After these preliminary analyses, data on subjects missing more than one multiplication open sentence or more than one division open sentence on the BMDT were set aside in order that the multivariate tests could be performed on subjects classified as competent in multiplication and division.

The Instructions to Subjects

In an investigation of this sort it was imperative that the subjects be familiar with the symbols involved. Any new or unfamiliar notation needed to be explained clearly. In addition, the directions needed to be clear and the set of numbers the students were to work with needed to be emphasized. In order to accomplish the above, an introductory cover page was designed (see Figure 2.1). All students had the same material on their cover pages. The test administrator verbally worked through the cover page with each group that participated in the investigation.

The test administrator discussed with the students what was meant by a "number puzzle." The test administrator asked what the following symbols represented: "x," "÷," and "=." The last symbol was introduced as "box" (■). It was explained to the students that this symbol was to be considered as a box and that there may or may not be a whole number hiding under the box. They were to decide whether or not there was a whole number under the box. If they could think of such a whole number, they were to write it on the line at the right of the number puzzle. If they could not think of any such whole number, they were to write "N" on the line at the right of the number puzzle to indicate that no such whole number existed.

The test administrator asked the students what whole number was hiding under the box between 7 and 9, between 10 and 12, between 15 and 17, and under the box following 20. It was emphasized that the whole numbers included the set {0, 1, 2, 3, 4, . . .}. The students

MY NAME IS ______________________________ Male/Female

SCHOOL ________________________ Math Teacher ________

GRADE ________

NUMBER PUZZLES

x ÷ = ■

0	1	2	3	4	5	6	7	■	9
10	■	12	13	14	15	■	17	18	19
20	■	..	..						

THE SET OF WHOLE NUMBERS: { o, 1, 2, 3, 4, . . . }

Decide which whole number is hiding under the box. Write the whole number hiding under the box on the line. If there is no whole number, write N on the line.

a. 55 < ■ < 57 ________

b. 72 < ■ < 74 ________

c. 99 < ■ < 100 ________

d. ■ x 1 = 6 ________

e. 1 ÷ ■ = 7 ________

f. 8 = 0 x ■ ________

g. 6 ÷ 2 = ■ ________

Figure 2.1. Cover page for number puzzles tests.

worked sample puzzles "a through g" together with the test administrator. The test administrator read the puzzles aloud and the students offered verbal responses. None of the responses were acknowledged as correct or incorrect. The responses from the test administrator were consistently to the effect, "You decide what you think the correct answer is and write that answer on the line at the right of the puzzle." It was anticipated that in working example c. $(99 < \blacksquare < 100)$, many students would sense readily that the correct response was "N." The test administrator asked "How about $99\frac{1}{2}$?" If the students agreed that the answer $99\frac{1}{2}$ would be a satisfactory response to example c, the test administrator explained to the class that numbers with fractions attached as part of that number were not whole numbers. The students completed the sample puzzles "d through f," writing whichever response they thought was correct on the line at the right of the puzzle.

The Pilot Study

A pilot study was deemed necessary to answer the following questions:

1. Was a multiplication and division open sentence test too difficult or too easy for fourth- through sixth-grade level students?
2. Was a 28 item test an appropriate length, and approximately how long would it take the students to respond to 28 open sentences?
3. Were the test directions clear to the subjects?

The pilot study also gave the test administrator experience in administering this instrument to students.

The test (NPT and BMDT) was administered to a group of 22 fifth-grade female students in Madison, Wisconsin in February, 1974. Based on the performance of the fifth-grade students, the test was considered appropriate for the fourth through sixth grades. The test took most of the students about 10 minutes with the slowest subjects completing the test in about 20 minutes. Since the directions and introductory work required about 5 minutes, it was decided that to administer the test to an intact classroom one should allocate approximately 30 minutes. The pilot study results indicated the subjects followed directions and responded to the open sentences in an appropriate manner.

The results of this pilot study were summarized in Appendix D. The division open sentences with the placeholder in a position (both operation-left and operation-right) were answered incorrectly more often than any other open sentence type.

The Sample

It was desirable to have subjects selected from a population employing a uniform mathematics textbook series. This allowed the investigator to measure more accurately the population's opportunity to learn. Waukesha, Wisconsin was chosen for this investigation.

Waukesha is an upper middle class socioeconomic suburb of Milwaukee, Wisconsin. The community is very supportive of education and in turn has a fine school system. The elementary schools are organized around the "neighborhood schools" concept. Of the 26 elementary schools in the city system, eight were chosen to participate in this study. These eight schools were selected by the Waukesha math coordinator, the assistant math coordinator, and the investigator. The initial four schools were picked at random. The remaining four schools were selected in an attempt to balance the sample so that both large and small schools were represented as well as schools from higher- and lower-socioeconomic areas. These eight schools were considered representative of the city school system. At the time of the study, Elementary Mathematics by Harcourt, Brace and World, 1966, was the uniform mathematics textbook employed city wide.

Those 1298 fourth-, fifth-, and sixth-grade students in attendance the half day the test was administered in their school were given the test. Table 2.3 indicates the eight schools employed, the number of classrooms within each grade, and the number of subjects in each classroom.

Mathematical Background of the Children in the Sample

Two methods were employed in order to obtain some measure of the mathematical background of the children in the sample. One method employed was an examination of the textbooks the students used. The schools in this system have employed the Harcourt, Brace and World Mathematics Textbook series (1966). The investigator tabulated all the examples of open sentence types a - n found within the third- through sixth-grade textbooks (excluding word problems). Grade three was included in order to have a two-year measure for the fourth-grade

Table 2.3

Sample Composition by School and Grade

		Number of Pupils			
School	Class	Grade 4	Grade 5	Grade 6	All
Banting	1	28	31	29	290
	2	25	32	32	
	3	26	27	33	
	4	27	--	--	
Hadfield	1	27	23	29	163
	2	30	27	27	
Heyer	1	25	14	28	167
	2	12	29	29	
	3	--	--	30	
Quarry	1	15	10	16	41
Randall	1	29	20	25	151
	2	29	22	26	
Saratoga	1	23	28	27	236
	2	24	27	28	
	3	25	28	26	
White Rock	1	18	16	24	58
Whittier	1	13	16	8	192
	2	29	25	25	
	3	28	24	24	
Total		433	399	466	1298

students. The frequency of occurrence of each open sentence type is summarized in Table 2.4.

As one can observe from Table 2.4, the students had no textbook experiences with open sentence types *h*, *j*, *m*, and *n*. There were very few instances of operation-right open sentences. There were no experiences with open sentences having no whole number solution. Students had experiences mainly with open sentence types *a*, *k*, and *e*. Limited experiences were provided with sentence type *c*. Student experiences provided with the remaining (*b*, *d*, *f*-*j*, *l*-*n*) sentence types were negligible.

Excluding word problems, Table 2.4 indicates the maximum exposure the students might have had to each open sentence type as furnished by the textbook. There was no way of finding out whether the individual teachers employed these examples in the text. Also the teachers might have introduced supplementary experiences with these various open sentence types.

A second method used to obtain some measure of the mathematical background of the children in the sample was to have the teachers of each of the 53 classes participating in the study fill out a questionnaire indicating the amount of experience they thought their students previously had with each open sentence type. A sample copy of the teacher questionnaire is included as Appendix E. Table 2.5 indicates the results of the teacher questionnaire. In examining Table 2.5, several patterns are noticeable.

Table 2.4

Frequency of Occurrence of Open Sentence Types
As Found in Mathematics Textbooks
(Harcourt, Brace, and World)

Open Sentence Type	Grade 3	Grade 4	Grade 5	Grade 6
a) a x b = ■	x	x	x	x
b) ■ = a x b	0	2	7	7
c) a x ■ = c	35	35	45	60
d) c = a x ■	0	0	0	12
e) ■ x b = c	187	219	254	125
f) c = ■ x b	1	60	0	22
g) ■ ÷ b = c	0	12	2	6
h) c = ■ ÷ b	0	0	0	0
i) a ÷ ■ = c	0	27	4	11
j) c = a ÷ ■	0	0	0	0
k) a ÷ b = ■	x	x	x	x
l) ■ = a ÷ b	0	0	4	3
m)* ■ x 9 = 29	0	0	0	0
n)# 3 = 28 ÷ ■	0	0	0	0

x--The majority of experiences and drill are involved with open sentence types a and k. For this reason the experiences for these two categories were not tallied. Excluding word problems, this table indicates the maximum exposure the students might have had to each open sentence type as furnished by the textbook. Numbers were used in examples m and n to distinguish no whole number solution examples in which c is not a basic fact product (as in m) and examples in which a is a basic fact product (as in n).

*--The four multiplication open sentence types with no whole number solutions are grouped together and represented by example m.

#--The four division open sentence types with no whole number solutions are grouped together and represented by example n.

Table 2.5

Student Experience With Each Open Sentence Type

(Rated by Teacher Opinion Questionnaire)

	Sentence Type	Exemplar	Grades 4th	 5th	 6th
Solution Exists in W	a. a x b = ■	7 x 8 = ■	2.72	2.69	3.00
	b. ■ = a x b	■ = 5 x 6	2.33	2.56	2.53
	c. a x ■ = c	6 x ■ = 42	2.00	2.31	2.33
	d. c = a x ■	35 = 5 x ■	1.83	2.12	2.33
	e. ■ x b = c	■ x 9 = 63	2.17	2.38	2.40
	f. c = ■ x b	40 = ■ x 8	1.94	2.06	2.47
	g. ■ ÷ b = c	■ ÷ 9 = 8	1.61	1.81	2.20
	h. c = ■ ÷ b	6 = ■ ÷ 8	1.33	1.69	1.87
	i. a ÷ ■ = c	30 ÷ ■ = 5	2.06	2.19	2.53
	j. c = a ÷ ■	6 = 42 ÷ ■	1.61	1.94	2.00
	k. a ÷ b = ■	56 ÷ 7 = ■	2.56	2.62	2.93
	l. ■ = a ÷ b	■ = 35 ÷ 7	2.00	2.31	2.47
No Solution Exists in W	m. ■ x b = c	■ x 9 = 29	1.39	1.31	1.60
	n. c = a ÷ ■	3 = 28 ÷ ■	1.39	1.38	1.60

Note. Scores were out of a total possible of three. A score of 1 represented little experience, 2 indicated moderate experience, and 3 indicated extensive experience.

1. Excluding a, b, m, and n, the experience scores increased from grade 4 to 5 and 5 to 6. The downward differences in a, b, m, and n were nearly negligible, being only .03, .03, and .08, and .01 respectively.
2. Open sentence types a and k were the open sentence types used for the majority of the multiplication and division sentences within the students' textbooks. These two open sentence types also had the highest experience scores as rated by the teachers.
3. Open sentence types m and n were not represented in the students' textbooks. With one small exception in the fourth-grade, these two open sentence types received the lowest experience scores.
4. In all but two cases, the ratings for the operation-left sentences were higher than the ratings for the operation-right sentences.

One needs to be cognizant of these indicators of students' opportunity to learn as one proceeds through this study. If the subjects' responses showed significant differences between the various sentence types, one should ask whether the differences were due to intrinsic difficulty differences between the various sentence types, students' opportunity to learn, or to a combination of both.

Assignment of Tests to Subjects

The four tests were printed in eight forms. The forms were balanced so that four forms had the NPT first followed by the BMDT. The remaining four forms had the reverse sequence, that is, the BMDT first followed by the NPT. The tests were passed out randomly to students. Since the tests were color coded, it was quite easy to arrange it such that no two people sitting close together had exactly the same form of the test. Therefore, copying should not have been a factor.

Test Administration

All the fourth-, fifth-, and sixth-grade students in the eight schools selected were tested in the four-day period, May 7th, 8th, 9th, and 10th, 1974. The schedule was arranged in such a way that the test administrator was in each of the eight schools a specified half day within that four-day period. Testing usually started mornings at 8:30 and afternoons about 1.

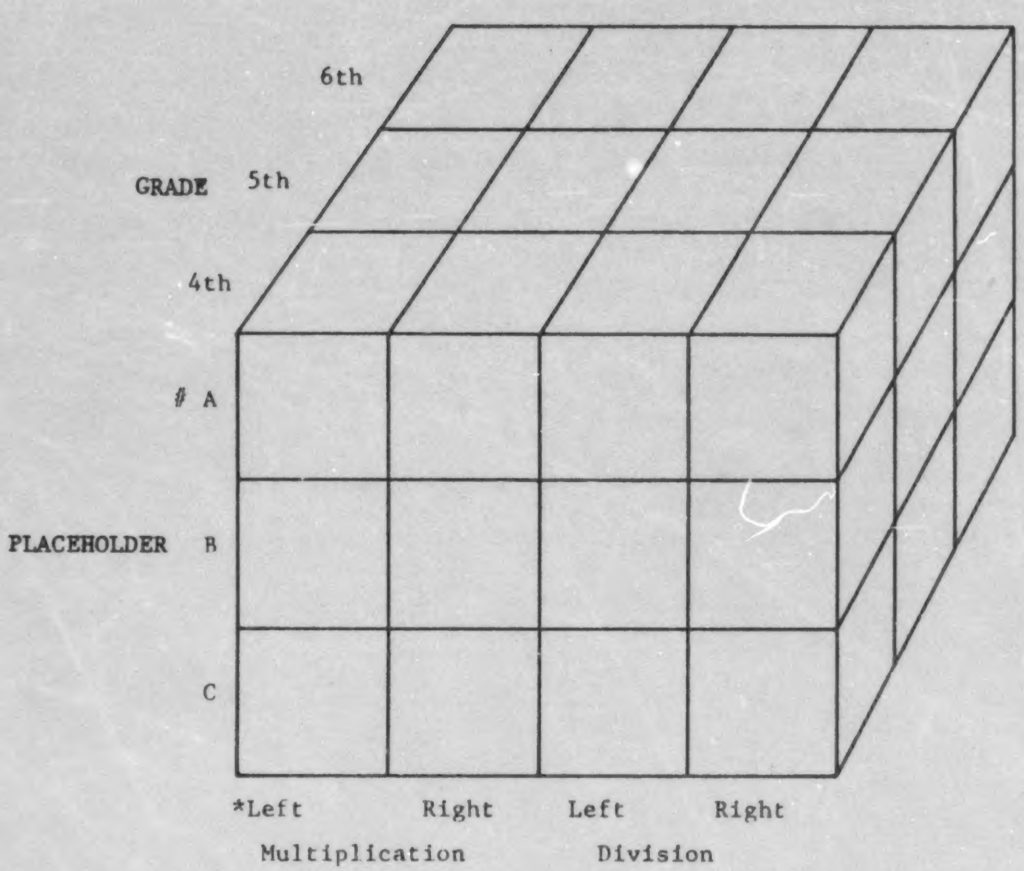
Two methods were employed to administer the tests. In both situations, the same test administrator distributed the tests and worked the introductory cover page with all the students. In situations where time permitted, the test administrator remained with the students until they had completed the tests. Within the larger schools, several classes had to be tested within the half day allocated. In some situations in which time did not permit the test administrator to remain in each classroom until the students had finished, the classroom teacher finished administering the tests. The tests were all collected at the end of the half day of testing.

The Experimental Design

The design used in this study was a 3 x 3 x 2 x 2 factorial design. These represented three grade levels, fourth, fifth, and sixth, respectively, three placeholder positions, _a_, _b_, and _c_, two operations, multiplication and division, and two symmetric forms, operation-right, and operation-left. A representation of the design is given in Figure 2.2.

Data Analysis

The data was corrected and coded by the investigator. The information was then key punched for computer analysis. The data was analyzed by the Fortap, DSTAT 2, and Finn Multivariate (Finn, 1967) package programs. The Wilcoxen Signed-Ranks Test was utilized to examine differences between responses to the open sentences with no whole number solutions.


6th
GRADE
5th
4th
# A
PLACEHOLDER
B
C
*Left
Right
Left
Right
Multiplication
Division

* Operation-left means the operation is on the left of the equality sign (a o b = c). Operation-right means the operation is on the right of the equality sign (c = a o b).

# Placeholder a is always to the left of the operation sign, b is to the right of the operation sign, and c is on the opposite side of the equality sign from the operation symbol.

Note - set W = {0, 1, 2, 3, 4, 5, 6, 7, . . .}

Figure 2.2. Representation of the experimental design.

Chapter III

DESCRIPTIVE DATA ANALYSIS

Chapter III is a presentation and discussion of the descriptive data analysis. The test was administered to 1298 subjects from grades four through six. The distribution of students by grade and test form is shown below in Table 3.1.

Table 3.1

Number of Students Per Test Form

Test	Form	Grade			
		4	5	6	All
1	A and B	113	102	113	328
2	C and D	116	103	118	337
3	E and F	103	101	116	320
4	G and H	101	94	118	313
	Total	433	400	465	1298

As Table 3.1 indicates, the number of students assigned to each of the four tests is relatively proportional.

The NPT was formed from a pool of 112 open sentences. The open sentences were divided among the four tests, thereby yielding 28 open sentences for each test. Considering the possible combinations of 3 placeholders, 2 operations, and 2 symmetric forms, 12 open sentences types can be generated. Each distinct open sentence type was assigned to exactly one cell of a 12 cell matrix (see Table 3.2).

Table 3.2

Open Sentence Types Assigned to Each of 12 Cells for Number Puzzle Test

Placeholder Position	CELL NUMBER			
	Multiplication		Division	
	Operation Left	Operation Right	Operation Left	Operation Right
a	1	4	7	10
b	2	5	8	11
c	3	6	9	12

For example, Table 3.2 indicates the multiplication operation-left open sentences with the placeholder in the c position were assigned to cell 3.

There were nine open sentences in cells 1, 2, 4, 5, 7, 8, 10, and 11. Cells 3, 6, 9, and 12 each contained 10 open sentences. In creating the open sentences with no whole number solutions, two open sentences were randomly selected from each cell. Recall that open sentence within cells 3, 6, 7, and 10 could not be altered to yield open sentences with no whole number solutions. Therefore the number of open sentences within these four cells remained unchanged. The resultant number of open sentences per cell is indicated in Table 3.3.

Table 3.3

Number of Open Sentences Assigned To Each Cell on Number Puzzle Test

Cell	Number of Whole Number Solution Open Sentences	Number of *No* Whole Number Solution Open Sentences
1	7	2
2	7	2
3	10	0
4	7	2
5	7	2
6	10	0
7	9	0
8	7	2
9	8	2
10	9	0
11	7	2
12	8	2

As Table 3.3 indicates, the number of whole number solution open sentences varies from seven to ten per cell. The no whole number solution cells either contain two open sentences or are empty.

Each student's BMDT and NPT was corrected by the investigator. The responses on the BMDT were marked either correct or incorrect. The responses on the NPT were coded with a number 0 through 7. The zero code indicated a blank or illegible answer. Code 1 indicated a correct response. Code 2 was employed when the subject responded with "N" to open sentences which had whole number solutions. Code 3 was used when the subject employed the inverse operation. The subject appeared to have multiplied (divided) while attempting to solve a division (multiplication) open sentence. For example, code 3 was used when the subject responded 45 to the open sentence 15 ÷ 3 = ■. Code 4 was used when the subject responded with a number quite close to the correct solution. For example, 5 x 7 = ■ was coded as 4 when subjects responded numbers from 28 to 42 exclusive of 35. Code 5 was used when subjects performed the specified operation "across the equality sign." For example, the subject responded 128 to the open sentence 8 x ■ = 16, indicating the subject might have considered the open sentence as 8 x 16 = ■. Code 6 was used to indicate the subject responded with some numeral not falling into any of the above categories. Code 7 was used when the subject performed the inverse operation "across the equality sign." For example, the open sentence 15 ÷ ■ = 3 received 45 as the response, indicating the subject might have considered the open sentence as 3 x 15 = ■.

After each NPT response was coded with a number 0-7, the resulting data was analyzed by the Fortap Statistical Package (Baker, 1969). Appendix F summarizes the response of all students who completed the NPT.

The results of the analysis are reported in two parts. The first part discusses the results for the whole number solution cells. The second part discusses the results of the no whole number solution cells.

Whole-Number Solution Cells

The percentages of responses coded 1, 2, and 5 are summarized in Tables 3.4 to 3.6. Each table summarizes the percentage of responses to each open sentence that received one particular code. Table 3.4 summarizes the percentage of code 1 responses. Code 1 indicated a correct response was given. As Table 3.4 indicates, the percent of students responding correctly to the open sentences in cell 1 varied from 85 percent to 96 percent. This percentage was fairly representative of all the cells, excluding cells 7 and 10 in which the performance level was very low. The highest percentage of correct responses for any one cell was 97 percent.

Table 3.4

Percent of Code 1 Responses on Number Puzzle Test

Cell	1	2	3	4	Example 5	6	7	8	9	10
1. ■ x b = c	87	95	85	93	96	86	92	--	--	--
2. a x ■ = c	93	94	95	94	93	93	96	--	--	--
3. a x b = ■	97	90	96	86	90	94	88	95	92	93
4. c = ■ x b	87	87	90	95	91	86	84	--	--	--
5. c = a x ■	85	93	80	86	97	90	90	--	--	--
6. ■ = a x b	92	92	94	96	93	94	93	96	97	94
7. ■ ÷ b = c	37	49	45	51	38	41	56	44	51	--
8. a ÷ ■ = c	91	81	90	92	85	90	95	--	--	--
9. a ÷ b = ■	84	80	88	91	90	85	86	91	--	--
10. c = ■ ÷ b	27	31	35	30	28	34	38	27	37	--
11. c = a ÷ ■	69	86	88	88	83	87	87	--	--	--
12. ■ = a ÷ b	89	87	83	87	78	80	89	81	--	--

Code 1 indicated the correct solution was given to the open sentence.

Code 2

Table 3.5 summarizes the percent of code 2 responses. The subject responded "N" when he thought there was no whole number solution. If the open sentence did have a whole number solution, an "N" response was coded "2." For the six multiplication cells, the frequency of this response was low, varing from 0 percent to 7 percent. However, observe the response patterns for cell 7. Table 3.5 indicates the percent of code 2 responses within cell 7 was 7 percent for one open sentence and varied from 38 percent to 54 percent for the remaining open sentences within that cell.

Code 3

Code 3 (i.e., the student performed the inverse operation) is not applicable for cells 1, 2, 4, 5, 7, 8, 10, and 11. Generally, code 3 was used infrequently. Appendix G summarizes the percent of code 3 responses given.

Code 4

No unusual or apparent patterns were indicated by examining the frequency of code 4 responses.

Code 5

Code 5 indicated the student apparently performed the given operation across the equality sign. For example, the open sentence 8 x ■ = 16 received 128 as the correct solution, indicating the subject might have considered the sentence to be of the type

Table 3.5

Percent of Code 2 Responses on Number Puzzle Test

		Example									
	Cell	1	2	3	4	5	6	7	8	9	10
1.	■ x b = c	5	1	5	3	0	6	3	--	--	--
2.	a x ■ = c	2	2	0	1	1	1	1	--	--	--
3.	a x b = ■	1	2	1	1	2	0	1	0	0	0
4.	c = ■ x b	4	5	4	0	3	7	4	--	--	--
5.	c = a x ■	6	1	7	5	0	3	2	--	--	--
6.	■ = a x b	3	1	3	2	2	1	2	2	1	1
7.	■ + b = c	51	7	46	39	54	53	39	44	38	--
8.	a + ■ = c	5	9	4	3	5	4	1	--	--	--
9.	a + b = ■	1	9	3	2	2	0	1	3	--	--
10.	c = ■ + b	61	55	55	59	62	59	9	64	52	--
11.	c = a + ■	14	5	5	6	8	6	7	--	--	--
12.	■ = a + b	5	7	8	4	13	7	5	7	--	--

Code 2 indicated the subject responded with "N," indicating no whole number solution existed.

$8 \times 16 =$ ■. This code was not utilized in cells 3, 6, 9, and 12 since it was not applicable within these cells. In each of these cells both numbers given are on the same side of the equality sign and therefore it is not feasible to talk about multiplying or dividing across the equality sign. Likewise, code 5 is not applicable for cells 8 and 11. In these cells, if the larger number on one side of the equation is divided by the smaller number on the other side of the equality sign, the resultant will be the correct answer. For example in cell 8, one open sentence was $18 \div$ ■ $= 3$. If the student divided 18 by 3, the resultant 6 would be the correct solution for that open sentence. In that event, the response would have been coded 1. There is no way of determining in this investigation how many students did the above process as opposed to the thought process similar to "What number can I divide into 18 and get 3 for the resultant?"

Within cells 1, 2, 4, and 5, Table 3.6 indicates the percent of responses coded as 5 ranged from 0 percent to 3 percent. Zero percent to 39 percent of the responses in cell 7 were coded 5. Eight of the open sentences had no responses coded as 5. One open sentence had 39 percent of the responses coded 5. As Table 3.5 indicates, eight of these nine open sentences in cell 7 received code 2 from 38 percent to 54 percent of the time.

The pattern of responses in cell 10 was basically the same as in cell 7. Zero percent responded with an answer coded 5 except for one open sentence which received 50 percent code 5 responses.

Table 3.6

Percent of Code 5 Responses on Number Puzzle Test

Cell		Example 1	2	3	4	5	6	7	8	9	10
1.	■ x b = c	0	0	0	0	1	0	0	--	--	--
2.	a x ■ = c	0	0	0	0	0	2	0	--	--	--
4.	c = ■ x b	0	0	0	0	0	0	0	--	--	--
5.	c = a x ■	0	1	0	0	0	0	3	--	--	--
7.	■ ÷ b = c	0	39	0	0	0	0	0	0	0	--
10.	c = ■ ÷ b	0	0	0	0	0	0	50	0	0	--

Code 5 indicated the subject performed apparently the operation across the equality sign. For example, the open sentence 8 x ■ = 16 received 128 as the correct solution, indicating the subject might have changed the sentence to the type 8 x 16 = ■.

The other open sentences within cell 10 received code 2 from 52 percent to 64 percent of the time, while one open sentence was marked code 2 only 9 percent of the time.

No Whole-Number Solution Cells

The analysis of the no whole number solution cells data is summarized in Appendix H. Table 3.7 summarizes the code 1, 6, and 7 responses to the open sentences within the eight no whole-number solution cells. With the exception of one open sentence, Table 3.7 indicates the correct response (code 1) was given from 78 percent to 89 percent of the time.

Responses coded 6 indicated the subject responded with some number that did not fall into any of the categories 1-5 or 7. The percent of code 6 responses varied from 3 percent to 13 percent.

Code 7 indicated the subject performed the inverse operation across the equality sign. This code was not applicable for cells 1, 2, 4, and 5 since if the student did perform the inverse operation across the equality sign, he would have derived the correct solution, i.e., "N." Similarly, code 7 was not applicable for cells 9 and 12, since both numbers were on the same side of the equality sign, which therefore precluded performing the operation across the equality sign.

For three of the four open sentences in cells 8 and 11, a total of five responses were coded 7. Table 3.7 indicates, however, that for the second open sentence in cell 11, 30 percent of the responses were coded 7.

Table 3.7

Percent of Code 1, 6, & 7 Responses for the No Whole-Number Solution Open Sentences on Number Puzzle Test

Cell	Percent of Code 1 Responses Examples 1	2	Percent of Code 6 Responses Examples 1	2	Percent of Code 7 Responses Examples 1	2
1. ■ x b = c	89	82	3	11	0	0
2. a x ■ = c	78	82	13	5	0	0
4. c = ■ x b	85	84	8	6	0	0
5. c = a x ■	86	82	6	9	0	0
8. a ÷ ■ = c	84	82	12	12	0	0
9. a ÷ b = ■	86	81	5	9	0	0
11. c = a ÷ ■	78	60	9	6	1	30
12. ■ = a ÷ b	81	82	12	7	0	0

Summary Conjectures Based on the Data Analysis

Code 1 responses were rather clear cut. Except for cells 7 and 10, the students' performance was rather consistent and the achievement levels were high.

Recall that code 2 was used when the subject responded "N" to open sentences to which he thought there was no whole number solution. The response pattern for cell 7 is unique. Seven percent responded "N" to $\blacksquare \div 2 = 8$. For the remaining eight open sentences within cell 7, the percent varied from 38 percent to 54 percent. Two questions are warranted.

First, why did only 7 percent respond "N" to $\blacksquare \div 2 = 8$ when such a high percent responded "N" to the other open sentences within the same cell? The answer to this question hinges on code 5--"subject performed the operation across the equality sign." Subjects did not make a code 5 response to any of the other open sentences in cell 7. None of the other open sentences in cell 7 involved numbers that enabled one to divide them and result in a whole number quotient. It would appear that the students saw the numbers 2 and 8 and the division sign and responded 4. The subjects either disregarded the position of the equality and operation signs, or mentally interchanged them for convenience.

A second question asks whether any cell other than cell 7 received such a high percent of "N" responses. Table 3.5 indicates cell 10 also had a high percent of "N" responses. The same pattern of responses occurred in cell 10 as occurred in cell 7. $6 = \blacksquare \div 2$

received responses coded 5 by 50 percent of the subjects. No other open sentence in cell 10 received responses coded as 5. Similarly, as in cell 7 no other open sentence in cell 10 involved digits that could be divided and result in a whole number quotient. With the exception of this one open sentence, the percent of "N" responses varied from 52 percent to 64 percent.

One might make two conclusions. First, in responding to division open sentences (both operation-left and operation-right) in which the placeholder is in position *a*, students consistently responded "N," indicating they thought the open sentence had no whole number solution. Secondly, when the division open sentence involved digits that could be divided and result in a whole number quotient, a high percent of students divided the two given numbers despite the relative positions of the equality sign, placeholder, and operation sign.

Code 3 (i.e., the student performed the inverse operation) was applicable only to cells 3, 6, 9, and 12. Generally, code 3 was used infrequently. With respect to multiplication, it was used in two open sentences in cell 3 (3 x 6 = ■ and 2 x 6 = ■) and two open sentences in cell 6 (■ = 6 x 2 and ■ = 4 x 2). Notice that in these open sentences one could divide the two numbers given and arrive at a whole number quotient. None of the other multiplication open sentences had this property.

With respect to division, few responses were coded 3. Other than two open sentences in cell 9, not more than five subjects used the inverse operation for any particular open sentence. Two open sentences in cell 9, namely, 10 + 5 = ■ and 8 + 4 = ■ , were answered by 15 and 20 subjects respectively with responses coded 3. This might be attributable to the size of the numbers involved in these two open sentences. 10 x 5 and 8 x 4 are number combinations quite familiar to many students. The larger numbers in the other open sentences might have caused the students to examine the open sentence more carefully since the product of the two numbers might not have been obvious.

Code 4 indicated the student responded with a number quite close to the correct number response. Within cell 3, the two open sentences 7 x 9 = ■ and 6 x 9 = ■ received the highest percentages of code 4 responses. This might imply the students knew about what the answer was, but could only come close to stating the correct answer. The fact that these two open sentences both involved nines along with the tendency for students to learn the nines table after all the others have been learned, might offer a partial explanation for these higher percentages of code 4 responses. Within cell 4, 6 percent responded 7 or 9 to the open sentence 56 = ■ x 8. Cell 5 had the highest percent of code 4 responses with 10 percent responding 2 or 4 to the open sentence 18 = 6 x ■ .

Within the division open sentences, cells 7 and 10 were extremely low with percent of code 4 responses ranging from 0 percent to 1 percent. Infrequent use of code 4 responses in these two cells might have been attributable to the high percent of responses coded 2 as described earlier. For the remaining four cells, the percent of responses coded 4 ranged from 0 percent to 7 percent. No unusual or apparent patterns were indicated by examining the frequency of code 4 responses within these four cells.

Code 5 indicated the student performed the specified operation across the equality sign. Within cells 1, 2, 4, and 5 (Table 3.6), the percent of responses coded as 5 ranged from 0 percent to 3 percent. The students generally multiplied across the equality sign only in open sentences in which the numbers involved were small. For example, ■ x 5 = 10, 2 x ■ = 8, and 12 = 4 x ■ are all examples in which the two numbers involved, when used as factors, were familiar to the students (i.e.,) 5 x 10, 2 x 8, and 4 x 12 were factors the students might have encountered previously). Students did not make responses coded 5 to open sentences that involved large numbers.

Within cell 7, 0 percent to 39 percent of the responses were coded 5. Eight of the open sentences had no responses coded as 5. One open sentence (■ ÷ 2 = 8) had 39 percent of the responses coded 5. As Table 3.5 indicates, eight of these nine open sentences in cell 7 received code 2 from 38 percent to 54 percent of the time. ■ ÷ 2 = 8 was marked code 2 only 7 percent of the time. ■ ÷ 2 = 8 was the only open sentence in cell 7 which involved two digits that could be divided and result in a whole number quotient. A possible conjecture for this pattern of responses might be as follows. The student confronted with the open sentence $\blacksquare \div 2 = 8$ sees 2, 8, and a ÷ sign. He thinks, $8 \div 2 = 4$ so "4" is the answer. He sees the remaining eight open sentence similar to $\blacksquare \div 7 = 4$ and sees 7, 4, and a ÷ sign. Whether he considers $7 \div 4$ or $4 \div 7$, neither results in a whole number. Therefore, the correct answer must be "N."

The pattern of responses in cell 10 was basically the same as in cell 7. Zero percent responded with an answer coded 5 except to the open sentence $6 = \blacksquare \div 2$, which received 50 percent code 5 responses. The other open sentences within cell 10 received code 2 from 52 percent to 64 percent of the time while this one open sentence was marked code 2 only 9 percent of the time. Similarly to the example in cell 7, $6 = \blacksquare \div 2$ was the only open sentence in which the two numbers could have been divided and result in a whole number solution.

Code 6 indicated the student responded with some number not falling into any of the other codes 1 through 5 or 7. Overall, the pattern of code 6 responses seemed to indicate there were fewer unexplained number answers for the multiplication open sentences than there were for the division open sentences.

Conjectures Based on the No Whole Number Solution Data Analysis

With the exception of the one open sentence, Table 3.7 indicates the correct response (code 1) was given from 78 percent to 89 percent of the time. The open sentence $3 = 7 \div \blacksquare$, received only 60 percent correct responses. Overall, the high percent of correct responses seemed to indicate the students could recognize open sentences with no whole number solutions and respond "N" to indicate no whole number solution existed.

Code 3 indicated the subject performed the inverse operation. No responses in cell 9 were coded 3. Within cell 12, a total of 13 students responded with answers coded 3. Performance of the inverse operation in cell 12 involved multiplying 20 x 3 and 25 x 4. These numbers may have been easier for students to multiply than the numbers in cell 9 (i.e., 13 x 4 and 41 x 8).

Between 2 percent and 10 percent of the responses were coded 4. Code 4 indicates the responses were quite close to the correct solution. For the open sentence $13 \div 4 = ■$, 7 percent of the responses were "3." Code 4 seemed to indicate the student gave the closest integral response to the correct rational number solution.

Code 5 was not particularly informative with respect to the open sentences with no whole number solutions.

Responses coded 6 indicated the subject responded with some number that did not fall into any of the categories 1-5 or 7. The percent of code 6 responses varied from 3 percent to 13 percent. The percent of code 6 responses given to the open sentences with no whole number solutions was noticeably higher than the percent of code 6 responses given to the open sentences which had whole number solutions.

Code 7 indicated the subject performed the inverse operation across the equality sign. For three of the four open sentences in cells 8 and 11, a total of five responses were coded 7. Table 3.7 indicates, however, that for the second open sentence in cell 11 (3 = 7 ÷ ■), 30 percent of the responses were coded 7. Since three of the four open sentences within cells 8 and 11 received only five responses coded 7, it seemed to indicate some reason existed to explain the increase to 30 percent of the students responding code 7 responses to the last open sentence within these two cells. The small numbers 3 and 7 utilized in the last open sentence might have been a possible factor contributing to the change of the pattern of responses. For the first three open sentences, to multiply across the equality sign would have required multiplying the following combinations: 35 x 9, 47 x 6, and 3 x 28. None of these combinations are among the basic facts. The last open sentence, however, becomes 3 x 7 when one multiplies across the equality sign. This is a basic fact and probably quite familiar to many of the subjects. Within the open sentences with no whole number solutions which involved larger numbers (28, 47, and 35), 78 percent to 84 percent of the students indicated no solution existed. In the open sentence involving small numbers (3 = 7 ÷ ■), only 60 percent responded correctly. Thirty percent responded "21." This seemed to indicate that number size probably did affect the pattern of responses given by students to these open number sentences.

Summary

The student performance levels for cells other than 7 and 10 were encouragingly high. When students were confronted with division, operation-left, placeholder in _a_ position puzzles, however, the performance levels decreased. It appeared that regarding these particular puzzles, students disregarded the position of the equality sign.

Student performance level on open sentences with no whole number solutions was remarkably high even though the students had no textbook experiences with these sentence types (see Table 2.4) and teachers predicted that students would do poorly on these sentence types (see Table 2.5). This seemed to indicate that students do not need to be instructed in each open sentence type in order to obtain high performance levels. The low performance levels attained in cells 7 and 10, however, indicated that attention needs to be given to division open sentences in which the placeholder is in the _a_ position.

Chapter IV continues this discussion, but further examines each factor through more precise statistical instruments. The first part of the chapter will discuss statistical differences among the open sentences having whole number solutions. The second part of the chapter will discuss statistical differences among the open sentences having no whole number solutions.

Chapter IV

STATISTICAL ANALYSIS OF THE DATA

After the descriptive statistics discussed in Chapter III had been generated from data furnished by all 1298 subjects, the subjects were separated into two categories. The first category consisted of students who either answered incorrectly or omitted no more than one multiplication open sentence and one division open sentence on the BMDT. The second category consisted of those students who did answer incorrectly or omit more than one multiplication open sentence or more than one division open sentence on the BMDT. All the data analysis discussed in this chapter utilized the data furnished by the subjects in the first category, i.e., those students who either answered incorrectly or omitted no more than one multiplication and one division open sentence on the BMDT. Since students in this category could respond correctly to multiplication and division open sentences in standard form, it seemed reasonable to attribute incorrect responses on the NPT to confusion with the particular open sentence type rather than to the number combination involved. Table 4.1 indicates by grade level the number of students in the first and second categories.

This chapter is organized in three main sections. The first section illustrates the overall plan for the data analysis. The second section discusses the analysis for open sentences having

Table 4.1

Number of Students by Grade and Test Form in the First and Second Categories

Forms	Grades						All Grades		All Grades Total in Categories 1 and 2
	4		5		6				
	First Category[a]	Second Category[b]	First Category	Second Category	First Category	Second Category	First Category	Second Category	
1 and 2	86	27	93	8	108	6	287	41	328
3 and 4	91	25	87	16	109	9	282	50	337
5 and 6	85	18	88	12	111	6	289	36	320
7 and 8	83	18	78	17	107	10	268	45	313
Total 1-8	345	88	346	53	435	31	1126	172	1298

[a] First category data used for further data analysis.

[b] Second category not used for further data analysis.

whole number solutions. The third section discusses the analysis for open sentences having no whole number solutions.

A. Plan for Data Analysis

The first analysis was done on the responses to whole number solution open sentences. The data generated by those students in the first category were analyzed by the DSTAT 2 Statistics Program (DSTAT 2:, 1973). This program summarized, for each of the 53 classes, the number of correct responses for each cell for each test form (see Figure 4.1--Step a) (Appendix J). Since there were four test forms, a mean was computed for each form for each cell which, when averaged produced the mean for the cell. Since there were one, two, or three puzzles on each test contributing to each cell, the cells on each test form contributing one puzzle were weighted by a factor of six, those with two were weighted by a factor of three, and those with three were weighted by a factor of two. The transformation matrix used is Appendix K. Table 4.2 indicates the transformed data matrix (see Figure 4.1--Step b).

The class means in the transformed data matrix were analyzed by ANOVA. Since analysis of variance only indicates whether significant differences exist and not how the factors differ, the following comparisons were made--grade level (4 vs. 5 vs. 6) multiplication versus division, operation-left versus operation-right, placeholder position (a vs. b vs. c)--and related interactions were investigated (see Figure 4.1--Steps c, d, e, f). Since the means in Table 4.2 are on a 0-6 scale, it was confusing to interpret exactly what means, for example, of 2.65 and 3.87

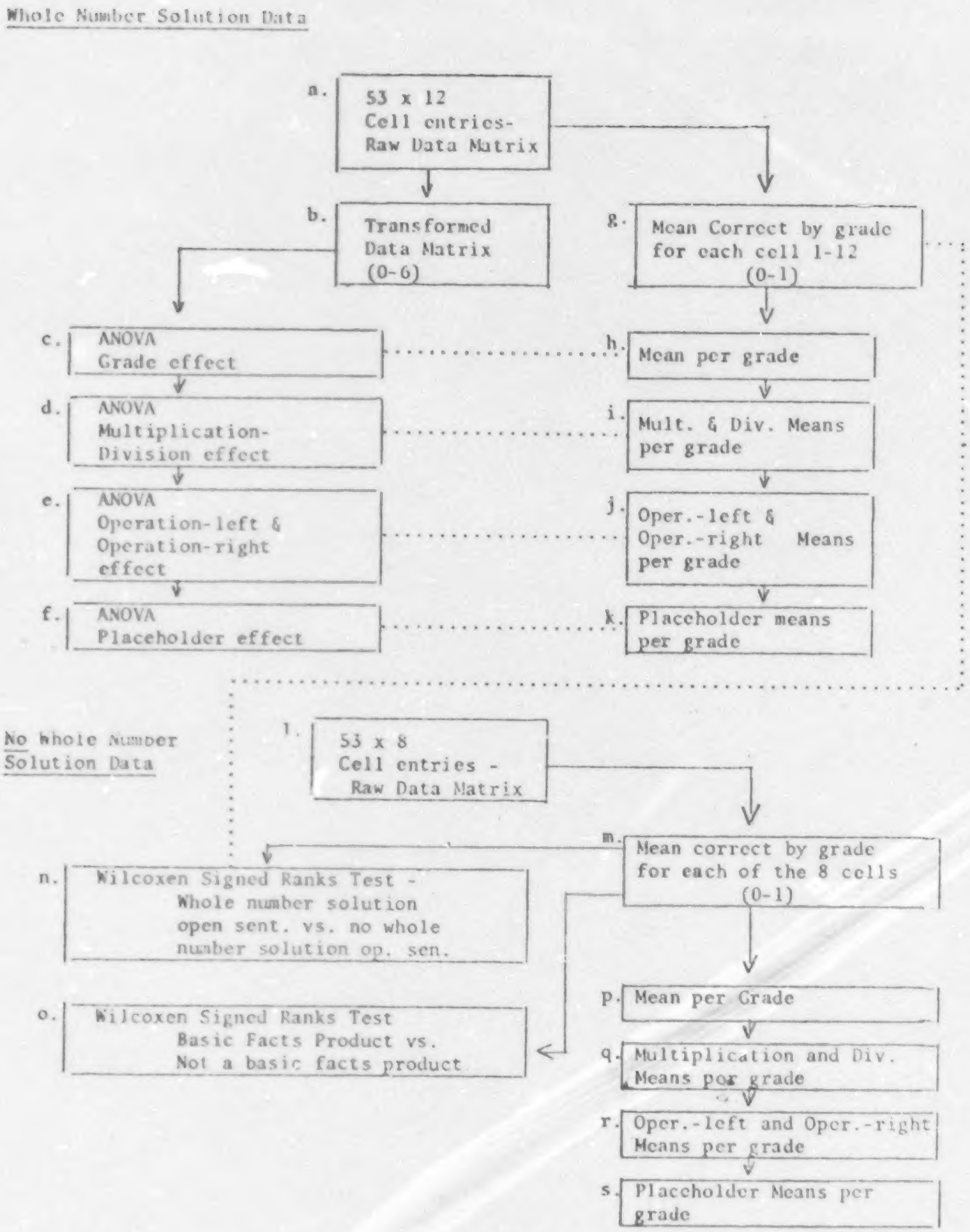

Whole Number Solution Data
a. 53 x 12 Cell entries- Raw Data Matrix
b. Transformed Data Matrix (0-6)
c. ANOVA Grade effect
d. ANOVA Multiplication-Division effect
e. ANOVA Operation-left & Operation-right effect
f. ANOVA Placeholder effect
g. Mean Correct by grade for each cell 1-12 (0-1)
h. Mean per grade
i. Mult. & Div. Means per grade
j. Oper.-left & Oper.-right Means per grade
k. Placeholder means per grade
No Whole Number Solution Data
l. 53 x 8 Cell entries - Raw Data Matrix
m. Mean correct by grade for each of the 8 cells (0-1)
n. Wilcoxen Signed Ranks Test - Whole number solution open sent. vs. no whole number solution op. sen.
o. Wilcoxen Signed Ranks Test Basic Facts Product vs. Not a basic facts product
p. Mean per Grade
q. Multiplication and Div. Means per grade
r. Oper.-left and Oper.-right Means per grade
s. Placeholder Means per grade

Figure 4.1. Schema for Data Analysis

Table 4.2

12 Class Means for Each of the 53 Classes--Used for ANOVA

Class	Cell 1	Cell 2	Cell 3	Cell 4	Cell 5	Cell 6	Cell 7	Cell 8	Cell 9	Cell 10	Cell 11	Cell 12
111	5.3626	5.9081	5.7272	5.9990	5.7272	5.7727	3.8635	5.4545	5.4999	1.6818	4.7726	5.3635
112	5.4285	5.9048	6.0000	5.4285	5.5237	5.7142	2.3808	5.5713	5.5713	1.1905	5.2857	5.1428
113	5.6521	5.9129	5.5216	5.8695	5.5216	5.7390	2.7825	5.8695	5.6087	1.3913	5.6086	5.6086
114	5.5788	5.5262	5.3157	5.5262	5.6315	5.6841	1.3683	5.5262	5.5788	1.3683	5.5262	5.4210
121	5.6537	5.7691	5.2307	5.7691	5.6153	5.6538	2.0384	5.6537	5.6922	1.1923	5.5384	4.8076
122	5.9258	5.9999	5.8147	5.8518	5.1480	5.6666	3.7777	5.7777	5.4443	1.3332	5.6666	5.4814
123	5.9129	5.9129	5.6956	5.9129	5.9999	5.6956	2.8696	5.8695	5.7390	2.0434	5.2173	5.7825
131	5.4999	5.8845	5.8076	5.6922	5.6922	5.8462	3.0000	5.7691	5.0769	2.3076	5.5384	5.3846
132	5.5861	5.7931	5.4137	5.6896	5.5861	5.6896	2.5172	5.5861	5.4137	2.0345	5.3792	5.3793
133	5.7812	5.6250	5.9062	5.5624	5.7499	5.8436	3.6250	5.9062	5.7812	2.5624	5.7186	5.6875
211	5.3683	5.5788	5.4210	5.5788	5.0525	5.0526	1.8421	5.2104	5.0526	0.7894	5.5262	5.1053
212	5.4000	5.7500	5.8500	5.1000	4.8500	5.8000	2.0650	5.4000	5.7000	1.2500	4.8000	5.3500
221	5.6818	5.7272	5.8636	5.7273	5.9090	5.5909	3.8182	5.8636	5.4545	3.2727	5.7272	5.6364
222	5.6799	5.9200	5.8400	5.3600	5.7600	5.9200	3.4400	5.5200	5.6400	2.2400	5.2800	5.3600
231	5.7777	5.8148	5.6296	5.5554	5.5926	5.9258	2.7407	5.7777	5.7777	1.5185	5.6666	5.4815
232	5.8888	5.7777	5.8148	5.5554	5.6296	5.8888	3.5926	5.4444	5.3333	2.9630	5.6666	5.5555
311	5.4736	5.8421	5.2105	4.9474	5.1053	5.4210	2.5263	5.6842	5.6316	1.3684	4.5789	4.2105

Table 4.2 (Continued)

12 Class Means for Each of the 53 Classes--Used for ANOVA

Class	Cell 1	Cell 2	Cell 3	Cell 4	Cell 5	Cell 6	Cell 7	Cell 8	Cell 9	Cell 10	Cell 11	Cell 12
312	6.0000	5.7273	5.7273	5.1818	5.6364	6.0000	1.8182	5.9999	5.5454	1.2727	4.9090	5.8182
321	5.9999	6.0000	5.7692	6.0000	5.8462	6.0000	4.1538	5.9999	4.9231	2.2308	5.7692	5.5385
322	5.9285	6.0000	5.6428	5.6786	5.7857	6.0000	3.8214	5.7857	5.8214	3.5714	5.5713	5.7143
331	5.3571	5.7857	5.7500	5.3214	5.8214	5.9286	3.1428	5.6785	5.1428	2.5714	5.7857	5.7143
332	5.9200	5.5200	5.6000	5.7600	5.6800	5.9200	3.2000	5.5200	5.3200	2.3200	5.5200	5.0400
333	5.7930	5.9310	5.8966	5.7931	5.9310	6.0000	4.1724	5.5861	5.9310	3.9655	5.7931	5.7931
411	5.8333	5.8333	5.7500	4.5000	5.3333	5.5000	2.9167	5.2500	5.3333	0.9167	5.0000	4.8333
421	5.4000	6.0000	5.7000	5.6000	5.4000	4.9000	4.0000	5.7000	4.8000	3.1999	4.8000	4.9000
431	5.9999	5.8750	6.0000	5.9999	5.6250	6.0000	2.6250	4.8750	6.0000	2.1875	5.4375	5.8125
511	4.9630	5.7407	5.7778	5.8519	5.5556	5.8519	2.7407	5.5556	5.4074	2.6667	5.2222	5.5556
512	5.8462	5.9231	5.6538	5.2308	5.6154	5.6923	3.1923	5.6538	5.6923	1.1923	5.0769	4.7692
521	5.4000	5.6000	5.6667	5.8667	5.8667	5.8000	3.6000	5.4000	4.8667	2.0667	5.6000	5.0000
522	5.9091	6.0000	5.8636	5.1364	6.0000	5.7727	3.4091	6.0000	5.8636	2.8182	5.7273	5.7273
531	5.9166	6.0000	5.6667	6.0000	5.7083	6.0000	3.0833	5.6250	5.9166	1.7917	5.6250	5.7500
532	5.7692	5.9231	5.5769	5.3462	5.6538	5.6923	3.5385	5.8846	5.7692	3.1923	5.7692	5.5769
611	4.5882	6.0000	5.8235	5.2941	5.7059	5.6471	2.4118	4.5882	5.1765	0.4706	3.8824	5.0588
612	5.3810	5.8571	5.2381	5.3810	5.7143	5.4762	1.5714	5.7143	5.0476	1.1429	5.0000	5.5714
613	5.8000	6.0000	5.8500	5.7000	5.4000	5.5500	2.6500	5.8500	5.5500	2.6000	5.1000	5.2500

Table 4.2 (Continued)

12 Class Means for Each of the 53 Classes--Used for ANOVA

Class	Cell 1	Cell 2	Cell 3	Cell 4	Cell 5	Cell 6	Cell 7	Cell 8	Cell 9	Cell 10	Cell 11	Cell 12
621	5.6923	5.9231	5.8846	5.2308	5.6538	6.0000	2.5000	5.6538	5.3846	1.2308	5.3077	5.5000
622	5.4545	5.8636	5.7727	5.7273	5.9091	6.0000	3.0000	5.5909	5.6364	1.6818	5.3182	5.1818
623	5.8261	5.7391	5.5652	5.5652	5.8261	5.8696	2.3043	5.4783	5.6522	1.0000	5.2174	5.3913
631	5.8695	5.7391	5.5217	5.7826	5.5217	5.9130	3.9130	5.8695	5.6087	3.0000	5.7391	5.6087
632	5.8929	5.6071	5.6071	5.6429	5.9286	5.9286	3.5714	5.5714	5.3571	3.3214	5.8929	5.7500
633	5.5454	5.5454	5.4544	5.6364	5.7273	5.8182	3.3636	5.5909	5.6364	1.5000	5.4544	5.3182
711	5.4999	4.6666	4.9999	3.4999	4.6666	4.6666	2.9999	3.4999	3.6666	0.0000	3.9999	3.4999
721	5.1110	4.6667	5.0000	3.8889	4.6667	5.3333	3.6667	5.0000	3.1111	2.1111	4.3333	4.2222
731	5.5237	5.9048	5.6190	4.2381	4.7143	5.6190	1.6667	5.1429	5.3810	1.5238	4.7143	4.7143
811	5.5385	6.0000	5.7692	5.3846	5.7692	6.0000	2.0769	5.7692	6.0000	0.9231	4.6154	5.6923
812	5.9999	5.6666	5.5414	6.5832	5.6666	5.8332	2.8750	5.6249	5.7083	1.2500	5.7499	5.4166
813	5.4399	5.6799	5.9200	5.6800	5.3599	5.9999	2.0400	5.7599	5.1599	1.3200	4.9199	5.1600
821	5.6874	5.9999	5.6250	5.6249	5.8749	6.0000	4.1249	5.6249	5.9999	3.3750	5.9999	5.8125
822	5.7142	6.0000	5.8095	5.9999	5.4286	5.4761	2.2381	5.5713	5.4286	1.6190	4.7142	5.4285
823	5.6666	5.9999	5.4443	5.4999	5.2778	5.7777	3.3333	5.3332	5.5556	1.4444	4.6666	4.7778
831	5.9999	5.9999	5.9999	5.9999	6.0000	5.7499	4.1250	5.6249	5.3750	4.3750	5.9999	5.6249
832	5.7390	5.7825	5.7390	5.2173	5.9130	5.6956	3.1738	5.9999	5.7826	2.5652	5.4782	5.2608
833	5.7618	6.0000	5.6667	5.7143	5.6189	5.7619	2.6667	5.7142	5.3333	2.3808	5.5713	5.6667

represented. Therefore, the percentage of correct responses for each of the 12 cells within each grade was computed. This was done to make the interpretation of the differences between and magnitudes of the means easier to compare. Percentages of correct responses are on a 1-0 metric within Tables 4.3 and 4.4. Within the discussion, means were converted to the more common 100-0 metric and are simply referred to as percentage correct.

Means were calculated by dividing the total number of correct responses given to each sentence type by the total number of possible correct responses (see Figure 4.1--Step g). These 36 means were used to calculate the overall means per grade, multiplication and division means, operation-left and operation-right means, and placeholder means (see Figure 4.1--Steps h, i, j, k).

A similar process was used for the no whole number solution open sentences. The DSTAT 2 Statistics Program summarized, for each of the 53 classes, the number of correct responses for each cell for each test form (see Figure 4.1--Step ℓ). Means, on a 1-0 scale, were computed by grade for each of the eight cells. These means were calculated by dividing the total number of correct responses given to each sentence type by the total number of possible correct responses (see Figure 4.1--Step m). Because some cells in the no whole number solution matrix were empty, something other than ANOVA was required to make comparisons with the whole number solution matrix. The Wilcoxen Signed Ranks Test was used to compare whole number solution open sentences with the no whole number solution open sentences (see Figure 4.1--Step n). The whole number solution means which were on a 1-0 were compared to the no whole number solution means which were also on a 1-0 scale. The Wilcoxen Signed Ranks Test was also used to compare responses to open sentences resulting from a basic fact product to those not resulting from a basic fact product (see Figure 4.1--Step o). The means by grade for each of the eight no whole number solution cells were used to calculate overall means per grade, multiplication and division means, operation-left and operation-right means, and placeholder means (See Figure 4.1--Steps p, q, r, s).

B. Open Sentences Having Whole-Number Solutions

In Table 4.3 the transformed means are presented, grade by grade, for each of the following factors and levels: (1) overall means, (2) operation multiplication compared to division, (3) operation-left compared to operation-right, and (4) means for the placeholder in a position, compared to b position, compared to c position. Each cell entry represents the mean of correct responses given by the students in the indicated grade. Table 4.4 indicates each cell mean by grade for the open sentences which have whole number solutions. For example, fourth grade--cell 1 (multiplication, operation-left, placeholder in a position) has a value of .9185. This indicates 91.85% of the fourth-grade subjects achieved the correct responses to the open sentences in cell 1 (MLA). The data in Tables 4.3 and 4.4 have been used to examine each of the following hypotheses. The formal analysis tables are contained in the appendix.

Table 4.3

Transformed Means of Whole-Number Solution Cells

Put on a 1-0 Metric

	Grade			
	4	5	6	all
Overall	.8062	.8556	.8652	.8423
Operation				
Multiplication	.9319	.9536	.9536	.9464
Division	.6805	.7576	.7767	.7383
Symmetric Property				
Operation-left	.8338	.8811	.8786	.8645
Operation-right	.7786	.8301	.8517	.8201
Placeholder position				
a	.6020	.7010	.7122	.6717
b	.9038	.9372	.9413	.9274
c	.9128	.9287	.9420	.9278

Placeholder position a was to the left of the operation sign, position b was to the right of the operation sign, and position c was on the opposite side of the equality sign from the operation sign. (a x b = c, or c = a ÷ b)

Table 4.4

12 Cell Means by Grade--

Whole-Number Solution Cells Put on a 1-0 Metric

	Cell #1 MLA		Cell #4 MRA		Cell #7 DLA		Cell #10 DRA
4th	.9185	4	.9168	4	.3679	4	.2049
5th	.9545	5	.9350	5	.5507	5	.3640
6th	.9582	6	.9346	6	.5342	6	.4220

	Cell #2 MLB		Cell #5 MRB		Cell #8 DLB		Cell #11 DRB
4th	.9574	4	.9049	4	.9143	4	.8386
5th	.9766	5	.9415	5	.9414	5	.8892
6th	.9684		.9342	6	.9332	6	.9292

	Cell #3 MLC		Cell #6 MRC		Cell #9 DLC		Cell #12 DRC
4th	.9452	4	.9487	4	.8997	4	.8576
5th	.9502	5	.9641	5	.9135	5	.8871
6th	.9530	6	.9733	6	.9249	6	.9169

M = multiplication, D = division

L = operation-left, R = operation-right

A = placeholder in a position, B = placeholder in b position

C = placeholder in c position, i.e., $a \times b = c$ and $c = a \div b$

1. H_o: The mean performance levels on open sentences having whole number solutions are the same for grade four (M_1), grade 5 (M_2) and grade 6 (M_3): $M_1 = M_2 = M_3$.

   H_1: Not H_0

Table 4.5 summarizes the Analysis of Variance of the data from Table 4.2.

Table 4.5

Analysis of Variance for Grade Level (4, 5, 6)

Source	DF	MS	F	P
Grade	2	7.9608	8.8211	.0006
Error	50	.9025		

Clearly, significant differences exist between grades with probability less than .0006. Therefore, the null hypothesis was rejected. The mean performance levels on open sentences which have whole number solutions was significantly different between grade levels. Table 4.3 indicates students overall performance increased across grades. Table 4.4 indicates every cell mean increased from grade 4 to grade 5. The cell means between grades 5 and 6 generally increased.

2. H_o: The mean performance level on open multiplication sentences (M_1) is the same as the mean performance level on open division sentences (M_2): $M_1 = M_2$

   H_1: $M_1 \neq M_2$

Table 4.6 indicates the results of the Analysis of Variance of the data from Table 4.2.

Table 4.6

Analysis of Variance for Operation (Mult., Div.)

Source	DF	MS	F	P
Operation	1	236.8298	1,333.0291	.0001
Error	50	.1777		

Clearly, significant differences exist between student performance levels on multiplication and division open sentences with $p < .0001$. The null hypothesis was therefore rejected. The mean performance level on open multiplication sentences was not the same as the mean performance level on open division sentences.

Table 4.3 indicates that within each grade the multiplication means were higher than the division means. Table 4.4 indicates that if the cells are compared pairwise with the operation-left or -right and placeholder variables both held constant, the multiplication means are higher than the division means in all pairs for all grades (i.e., cell 1 compared to cell 7, 4 to 10, etc.).

3. H_o: The mean performance level on operation-left open sentences (M_1) is equal to the mean performance level on operation-right open sentences (M_2): $M_1 = M_2$.

H_1: $M_1 \neq M_2$

Table 4.7 summarizes the operation-left and operation-right differences resulting from the Analysis of Variance of the data from Table 4.2.

Table 4.7

Analysis of Variance for Symmetric Property (Oper-Left, Oper-Rt.)

Source	DF	MS	F	P
Symmetric Property	1	12.2750	91.0433	.0001
Error	50	.1348		

Clearly, significant differences exist between the mean performance levels on operation-right and operation-left open sentences with $p < .0001$. The null hypothesis was therefore rejected. The mean performance level on operation-left open sentences was significantly different from the mean performance level on operation-right open sentences.

Table 4.3 indicates that, across all grades, the means for the operation-left open sentences were higher than the means for the operation-right open sentences. Table 4.4 indicates that if the cells are compared pairwise with operation and placeholder held constant within each pair, the operation-left means are higher in all pairs across all grades with the exception of cells 3 and 6. With the operation multiplication and placeholder in the _c_ position, student performance was slightly higher on operation-right open sentences. In all other cases, performance was higher on operation-left open sentence.

4. H_o: The mean performance levels on open sentences with the placeholder in position a (M_1), b (M_2), and c (M_3) are the same: $M_1 = M_2 = M_3$.

H_1: Not H_o

There were three placeholder positions tc examine. The first contrast examined was a vs. b. The second contrast examined was ab vs. c. Table 4.8 summarizes the placeholder position statistics resulting from the Analysis of Variance of the data from Table 4.2.

Table 4.8

Analysis of Variance for Placeholder Positions

(a vs. b and ab vs. c)

Source	DF	MS	F	P
Placeholder	2	164.1808	903.3201	.0001
a vs. b	1	249.8057	1325.8050	.0001
ab vs. c	1	76.5559	480.8352	.0001
Error	100	.1759		

(multivariate analysis used to test placeholder)

Clearly, significant differences exist between placeholder positions with $p < .0001$. The null hypothesis was therefore rejected. The mean performance levels on open sentences were not the same for placeholder positions a, b, and c.

Table 4.3 indicates the means, across all three grades, were the lowest for placeholder a. The means for the placeholder positions b and c were quite close in value. Table 4.4 indicates the means for the cells comprised of the division open sentences with placeholder in a position (both operation-left and operation-right) are much lower than any other group of cells. The means were slightly higher with operation-left cells, but overall considerably lower than all the other cell means.

The means for the four cells involving placeholder position b appear to be quite close in value to the means for the four cells involving placeholder position c. One might infer that performance on open sentences with the placeholder in the b position was about the same as performance on open sentences with the placeholder in the c position.

5. H_o: No interactions with $p < .05$ exist among the following factors: grade level, operation, symmetric property, and placeholder position.

   H_1: Interactions with $p < .05$ exist among the following factors: grade level. operation, symmetric property, and placeholder position.

Table 4.9 summarizes the results of interactions tested by Analysis of Variance of the data from Table 4.3.

Table 4.9

Analysis of Variance Interactions

Source	DF	MS	F	P
G x O	2	1.9948	11.2279	.0001
Error	50	.1777		
G x S	2	.8388	6.2213	.0039
Error	50	.1348		
G x P	4	1.3578	7.7182	.0012
Error	100	.1759		
O x S	1	5.0422	61.6801	.0001
O x S x G	2	.3106	3.7994	.0292
Error	50	.0817		
O x P	2	138.0383	667.4437	.0001
O x P x G	4	.7256	3.7465	NS
Error	100	.1946		
S x P	2	4.2760	57.1078	.0001
S x P x G	4	.0593	.9044	NS
Error	100	.1052		
O x S x P	2	2.2694	26.5586	.0024
O x S x P X G	4	.1158	1.6164	NS
Error	100	.0780		

G = Grade
O = Operation
S = Symmetric Property
P = Placeholder Position

Table 4.9 indicates the following interactions existed at a significance level of $p < .05$: (1) G x O, (2) G x S, (3) G x P, (4) O x S, (5) O x S x G, (6) O x P, (7) S x P, and (8) O x S x P. Since the analysis of variance indicated significant interactions existed at $p < .05$, the null hypothesis was rejected. Significant interactions existed among the following factors: grade level, operation, symmetric property, and placeholder position.

Table 4.4 indicates the means, across all three grades, were the lowest for cells 7 and 10. It would appear that very significant interaction occurred between operation division and placeholder position a.

C. Open Sentences Having No Whole-Number Solutions

Similar to Part B, the no whole number solution data generated by those students who either answered incorrectly or omitted no more than one multiplication and one division open sentence on the BMDT was analyzed by the DSTAT 2 Statistics program. With the results from this statistics program, the investigator was able to calculate the eight means (eight no whole number solution cells) for each class participating in the study. Notice cells 3, 6, 7, and 10 are empty. All open sentences of these types necessarily have whole number solutions. Table 4.10 indicates these cells in relation to the other cells.

Table 4.10

Cells for Each Grade

Placeholder	Multiplication *Op.-left	 Op.-rt.	Division Op.-left	 Op.-rt.
#A	1	4	X	X
B	2	5	8	11
C	X	X	9	12

*Operation-left means the operation is on the left of the equality sign (a o b = c). Operation-right means the operation is on the right of the equality sign (c = a o b).

#Placeholder _a_ is always to the left of the operation sign, _b_ is to the right of the operation sign, and _c_ is on the opposite side of the equality sign from the operation symbol.

Since 4 of the 12 cells for each grade were empty, an Analysis of Variance was not practical. The eight means for each class were averaged to produce means by grade for each cell. The results are indicated in Table 4.11. The means were utilized to form the more general means indicated in Table 4.12. As each hypothesis is examined, reference will be made to these two tables.

6. H_o: The mean performance level on open number sentences which have no whole number solutions (M_1), is equal to the mean performance level on open sentences which have whole number solutions (M_2): $M_1 = M_2$.

H_1: $M_o \neq M_1$

Table 4.11

Eight Cell Means by Grade
No Whole Number Solution Cells
Put on a 1-0 Metric

	Cell #1 MLA
4th	.8103
5th	.9091
6th	.8745

	Cell #4 MRA
4	.8103
5	.8908
6	.8874

	Cell #7 DLA
4	X
5	X
6	X

	Cell #10 DRA
4	X
5	X
6	X

	Cell #2 MLB
4th	.7661
5th	.8287
6th	.8858

	Cell #5 MRB
4	.8421
5	.8619
6	.9361

	Cell #8 DLB
4	.7874
5	.8970
6	.8571

	Cell #11 DRB
4	.6954
5	.7333
6	.6710

	Cell #3 MLC
4th	X
5th	X
6th	X

	Cell #6 MRC
4	X
5	X
6	X

	Cell #9 DLC
4	.8187
5	.8398
6	.9269

	Cell #12 DRC
4	.7953
5	.8398
6	.9087

M = multiplication, D = division

L = operation-left, R = operation-right

A = placeholder in _a_ position, B = placeholder in _b_ position,

C = placeholder in _c_ position, i.e., a x b = c, and c = a ÷ b.

X indicates no open sentence of this type exists, i.e., all open sentences of this type have whole number solutions.

Table 4.12

Means of No-Whole Number Solution Cells

Put on a 1-0 Metric

	Grade			
	4	5	6	All
Overall	.7907	.8500	.8684	.8364
Operation				
Multiplication	.8072	.8726	.8960	.8586
Division	.7742	.8275	.8409	.8142
Symmetric Property				
Operation-left	.7956	.8686	.8861	.8501
Operation-right	.7858	.8315	.8508	.8227
Placeholder Position				
a	.8103	.9000	.8810	.8638
b	.7228	.8302	.7926	.7819
c	.8070	.8398	.9178	.8549

Placeholder position a was to the left of the operation sign, position b was to the right of the operation sign, and position c was on the opposite side of the equality sign from the operation sign. (a x b = c, or c = a + b)

Several statistical tests could have been employed to determine whether statistically significant differences existed between the means for the no whole number solution cells and the means for the whole number solution cells. The Wilcoxen matched pairs signed ranks test was selected to test for significant differences between the two groups. This test has all the advantages of the sign test as well as taking into account the magnitude of the differences between rankings of the scores in the two distributions. Hays (1963) claims the Wilcoxen test has a very high power-efficiency compared to the other methods designed specifically for the matched-pair situation. Four separate Wilcoxen signed ranks tests were done. These tests were not independent because of pooling. The data were pooled originally to test for overall significance. Since the overall test showed significant differences existed, three separate tests were run independently, one at each grade level. The results of the Wilcoxen signed ranks tests are indicated in Table 4.13.

Table 4.13 indicates that, for the three grades combined, the probability was less than .0001. The probability was less than .0059 for grades four and five. The probability was less than .0250 for grade six. The Wilcoxen test indicated that with $p < .05$, significant differences existed between the means for the open sentences with whole number solutions and the open sentences with no whole number solutions. Since the probabilities for all three grades and the overall probability were all less than .05, the null

Table 4.13

Wilcoxen Test

Comparison of correct responses given to eight cells for which whole-number solutions existed and the same eight cells for which no whole-number solutions existed.

Put on a 1-0 Metric

	Means of 8 Cells Whole Number Solution Exists	Means of 8 Cells No Whole Number Solution Exists	Difference	Signed Rank	P and z Values
		4th Grade			
Cell 1	.92	.81	-.11	-4.5	
Cell 2	.96	.77	-.19	-8	
Cell 4	.92	.81	-.11	-4.5	
Cell 5	.90	.84	-.06	-1.5	$p < .0059$
Cell 8	.91	.79	-.12	-6	
Cell 9	.90	.82	-.08	-3	
Cell 11	.84	.70	-.14	-7	
Cell 12	.86	.80	-.06	-1.5	$\|z\| = 2.52$
		5th Grade			
Cell 1	.95	.91	-.04	-1.5	
Cell 2	.98	.83	-.15	-7	
Cell 4	.94	.89	-.05	-3.5	$p < .0059$
Cell 5	.94	.86	-.08	-6	
Cell 8	.94	.90	-.04	-1.5	
Cell 9	.91	.84	-.07	-5	
Cell 11	.89	.73	-.16	-8	
Cell 12	.89	.84	-.05	-3.5	$\|z\| = 2.52$
		6th Grade			
Cell 1	.96	.87	-.09	-7	
Cell 2	.97	.89	-.08	-6	
Cell 4	.93	.89	-.04	-4	$p < .0250$
Cell 5	.93	.94	+.01	+2	
Cell 8	.93	.86	-.07	-5	
Cell 9	.92	.93	+.01	+2	
Cell 11	.93	.67	-.26	-8	
Cell 12	.92	.91	-.01	-2	$\|z\| = 1.9608$

Three grades grouped together--$p < .0001$ ($|z| = 4.1714$)

hypothesis was rejected. The mean performance level on open number sentences which have no whole number solutions was significantly different from the mean performance level on open sentences which have whole number solutions.

Table 4.14 indicates both the _no_-whole-number-solution cell means taken from Table 4.12 and the whole-number-solution cell means taken from the comparable eight cells within Table 4.4. Table 4.14 shows that within each grade level, the mean for the eight whole number solution cells was greater than the mean for the eight _no_ whole number solution cells.

Table 4.14

Means by Grade for the 8 _No_-Whole-Number-Solution Cells Compared to the Comparable 8 Whole-Number-Solution Cells

	Means for the Comparable Eight Cells for Which Whole Number Solutions Exist	Means for the Eight _No_ Whole Number Solution Cells
Average for the Three Grades	.9228	.8364
4th Grade	.9010	.7907
5th Grade	.9298	.8500
6th Grade	.9374	.8684

The means for each cell (the eight no whole number solution cells and the comparable eight whole number solution cells) within each grade were rounded off to two significant digits and are listed in Table 4.13. Of the 24 differences between the whole number solution cells and the comparable no whole number solution cells, 22 were negative values and only two were positive. The students seemed to have a higher mean on open sentences which had whole number solutions than they had on open sentences which had no whole number solutions.

7. H_o: There is no significant difference between the mean performance level on multiplication (division) open sentences in which the product (dividend) is a product of some basic fact (M_1), and the mean performance level on multiplication (division) open sentences in which the "product" (dividend) is not a "product" of some basic fact (M_2): $M_1 = M_2$.

H_1: $M_1 \neq M_2$

In order to test the hypothesis, the eight no whole number solution cells were divided into two groups. The first group consisted of those in which the product (dividend) was a basic-fact product. The second group consisted of those in which the "product" (dividend) was not a basic-fact "product." The number of correct responses (transformed to a 1-0 metric) for each of the eight cells are displayed in Table 4.15.

Table 4.15

No Whole-Number-Solution Open Sentences

Basic Fact Products Compared to Not a Basic Fact Products

Number Correct Transformed to a 1-0 Metric

Wilcoxen Matched Pairs Signed Ranks Test

Cell	Number Correct Basic Fact Pro.	Number Correct- Not a Basic Fact Pro.	Difference	Rank	Signed Rank
1	.89	.82	-7	7	-7
2	.78	.82	+4	5	+5
4	.85	.84	-1	1	-1
5	.86	.82	-4	5	-5
8	.84	.82	-2	2.5	-2.5
9	.78	.60	-18	8	-8
11	.81	.85	+4	5	+5
12	.82	.84	+2	2.5	+2.5

$z = .77$
$p < .2206$ N.S.

The information was statistically analyzed by the Wilcoxen matched pairs signed ranks test. The analysis is demonstrated in Table 4.15. The mean number of correct responses to the "not a basic fact" open sentences was .80, while the mean number of correct responses to the "basic fact" open sentences was .83.

The probability resulting from the Wilcoxen analyses was less than .2206, which was not significant. The null hypothesis was therefore not rejected. There was no significant difference between mean performance on multiplication (division) open sentences in which the product (dividend) is a product of some basic fact and mean performance on multiplication (division) open sentences in which the "product" (dividend) is not a "product" of some basic fact.

Summary

Several factors appeared to influence performance levels.

1. Grade level affected performance levels. Fifth-grade students had a higher performance level than fourth-grade students and sixth-grade students performed at a slightly higher level than the fifth-grade students.
2. The performance level on multiplication open sentences was higher than on division open sentences.
3. The performance level on operation-left open sentences was higher than on operation-right open sentences.
4. The position of the placeholder appeared to influence the performance level.
5. The existence or nonexistence of a whole number solution influenced the performance level.

The analysis was complex to interpret because of the significant interactions. There appeared to be a very high interaction between operation division and placeholder position a. Significant interactions also existed between the following factors: (1) grade and operation; (2) grade and symmetric factor; (3) grade and placeholder position;

(4) operation and symmetric factor; (5) operation, symmetric factor, and grade; (6) operation and placeholder position; (7) symmetric factor and placeholder position; and (8) operation, symmetric factor, and placeholder position. Caution must be exercised, therefore, in taking an overly simplistic interpretation of significant differences between levels of principal factors.

Chapter V

CONCLUSION TO THE THESIS

Introduction

This chapter presents a summary of the study, discusses conclusions and implications resulting from the study and offers recommendations for future research.

Summary

The main purpose of this study was to find out whether differences exist in pupils' performance when solving open sentences in which the open sentence types were varied.

Specifically, this investigation sought to find out the differences in students' responses to open number sentences when the following factors were varied: (a) school grade [4, 5, and 6], (b) the symbol for the operation specified in a sentence [x and ÷], (c) sentence type as determined by the symmetric property of the equality relation [a o b = c versus c = a o b), (d) the position of the placeholder in a sentence [a, b, or c], (e) the existence or non-existence of an open sentence solution within the set of whole numbers [■ x b = 20 versus ■ x 5 = 21], and (f) the largest number being a basic fact product or not a basic fact product in open sentences which have no whole number solution [3 x ■ = 25 versus 3 x ■ = 23].

Two distinct kinds of multiplication and division open sentence tests were constructed and administered to 1298 fourth-, fifth-, and sixth-grade students from eight schools. Each student was administered a 28-item open sentence number puzzle test (NPT) and a 14-item basic multiplication and division test (BMDT).

Four forms of the NPT were constructed. Based on the above factors (operation, symmetric property, placeholder effect, and existence or non-existence of a whole number solution), 20 distinct open sentence types were identified for the NPT. Fifty-six multiplication and 56 division facts resulted from the assignment of the numerals two through nine to a and b (no doubles). These 112 number facts were partitioned into four groups of 28 items. Each group of 28 number facts was assigned to one of the four test forms and one of the 20 open sentence types.

Information gained when open sentence types are varied would be questionable if the students do not know the basic facts. To find students' performance level on operation-left, canonical form open sentences (e.g, 2 x 6 = ■), each student was given a basic multiplication and division test (BMDT). Each BMDT was composed of five multiplication facts, five division facts, and four open sentences involving the number one. To balance for the learning effect the first half of the test might cause, half the students responded to the NPT first, followed by the BMDT. The other half of the students responded to the BMDT first, and then the NPT.

The conclusions and implications resulting from this study can be stated with reasonable certainty only for the population from which the subjects were selected. The results are valid for students within the fourth-, fifth-, and sixth-grades within the Waukesha, Wisconsin city school system. The generalizability of the results to other students depends on the differences between them and the given population. For instance, students in different grades, students who had used a different mathematics textbook series, and students having experienced a different opportunity to learn could be enough unlike the students in this population that different results could be anticipated.

In order to obtain information concerning the subjects' opportunity to learn, two procedures were used. A questionnaire was administered to all the classroom teachers of students participating in the study. The individual teachers rated each type of open number sentence (20) as to the experience they believed their students had previously had with that particular type of open sentence. Also, a thorough examination was conducted of the textbooks the students used in grades three through six (Elementary Mathematics, Harcourt, Brace, & World, 1966). A table was constructed which indicated the maximum number of experiences the students might have had with each open sentence type as indicated by the textbooks. This summary did not include word problems. A careful examination of textbook experiences the students might have been exposed to, together with an indication from the teachers of experiences the students might have had with each type open sentence, offered an indication of the students opportunity to learn.

The examination of the textbooks indicated students experiences with these open sentence types was mostly limited to two types-- $a \times b = \blacksquare$ and $a + b = \blacksquare$. Open sentence types $\blacksquare \times b = c$ appeared occasionally. The other open sentence types appeared very infrequently or not at all. The teacher opinion questionnaire indicated parallel results. The two open sentence types the teachers predicted students would score highest on were a x b = ■ and a ÷ b = ■ . Similarly, the sentence types mentioned infrequently or not at all in the textbooks were scored lower than the other sentence types. The sentence types having no whole number solution were rated lower than all the whole number solution sentence types.

The data furnished by all 1298 subjects was corrected and coded by the investigator. The information was then key punched for computer analysis and analyzed by the Fortap Statistical Package. This process yielded descriptive statistical results reported in Chapter III.

After the descriptive statistics had been generated, the data were separated into two groups. The first group consisted of the data generated by those students who missed no more than one multiplication open sentence and one division open sentence on the BMDT. The second group consisted of the data generated by the students who did miss more than one multiplication open sentence or more than one division open sentence on the BMDT. All the remaining data analysis utilized those subjects in the first group.

The investigator calculated the means of the 12 whole number solution cells for each class participating in the study. These cell means were the raw data used for the analysis of variance.

Conclusions and Implications

Based on the reported results in Chapter IV, the following conclusions were drawn.

1. The performance level of subjects on open sentences having whole number solutions was significantly different between grade levels.

Students' level of performance was measured within the fourth, fifth, and sixth grades. Significant differences do exist between grades ($p < .05$). For the open sentences having whole number solutions, there was an increase in performance level across all three grades. Fourth-grade overall average was 80.62 percent, fifth-grade was 85.56 percent, and sixth-grade was 86.52 percent. Similar findings resulted for the open sentences having no whole number solutions. Fourth-grade overall average was 79.07 percent, fifth-grade was 85.00 percent, and sixth-grade was 86.84 percent.

These results are in agreement with the results from Weaver's (1971) study. In Weaver's study the performance level on open addition and subtraction sentences increased from grade 1 to grade 2 to grade 3. This seems to indicate that as the students have more experiences in mathematics and more opportunities to learn their performance level increases.

2. The performance level of subjects on open multiplication sentences was significantly different from the performance level of subjects on open division sentences.

Significant differences exist between student performance levels on open multiplication sentences and open division sentences ($p < .0001$). Within each grade, the multiplication means were higher than the division means. The average student performance level on multiplication open sentences with whole number solutions was 94.64 percent compared to

73.83 percent on division open sentences with whole number solutions. The student performance level on multiplication open sentences with no whole number solutions was 86.19 percent compared to 81.42 percent on division open sentences with no whole number solutions. This seems to clearly indicate students' performance level was higher on multiplication open sentences than on division open sentences.

These results are in parallel agreement with the results from Weaver's study (1971). In Weaver's study the performance level was higher for addition sentences than for subtraction sentences within each grade.

A partial explanation for the students higher performance level on multiplication open sentences might be the fact that students usually study multiplication facts before division facts. Therefore, they have had a greater opportunity to learn multiplication facts than division facts. A few of the fourth-grade classes either had not yet studied division or had had little exposure to division at the time the test was administered.

3. The performance level of subjects on operation-left open sentences was significantly different from the performance level of subjects on operation-right open sentences.

Significant differences exist between student performance levels on operation-left open sentences and operation-right open sentences ($p < .0001$). Examining responses to the whole number solution open sentences, students answered correctly 86.45 percent of the operation-left sentences and only 82.02 percent of the operation-right sentences. For the open sentences with no whole number solutions, students answered correctly 85.02 percent of the operation-left sentences and only 82.27 percent of the operation-right sentences. This seems to indicate students' performance level was higher on the operation-left open sentences than on the operation-right sentences.

These results are in agreement with the results from Weaver's study (1971). In Weaver's study, the performance level was consistently higher for the operation-left open sentences than for the operation-right open sentences.

Student opportunity to learn might account for part of the difference between the performance level on operation-left and operation-right open sentences. Nine hundred and eighty examples (excluding types a and k) within the third- through sixth-grade textbooks were operation-left, while only 148 were operation right. It would seem that the students have had more experiences with the operation-left open sentences than with the operation-right open sentences. If teachers want students to be able to solve operation-right open sentences as accurately as operation-left open sentences, it would appear that more experiences with operation-right open sentences are needed.

4. The performance level of subjects on open sentences was significantly different for placeholder positions _a_, _b_, and _c_.

Significant differences exist among the placeholder positions ($p < .05$). Contrasting placeholder *a* and *b* and *a* and *b* combined compared to *c*, significant differences exist with $p < .0001$. Since it was difficult to determine exactly what information was revealed by the contrast *ab* vs. *c*, it was decided to rerun the analysis of variance a second time, utilizing contrasts *b* vs. *c* and *bc* vs. *a*. By examining the output from both runs, information was revealed concerning the following contrasts--*a* vs. *b*, *ab* vs. *c*, *b* vs. *c*, and *bc* vs. *a*. The researcher believed an examination of contrasts *a* vs. *b* and *b* vs. *c* offered an accurate picture of the true situation. All the contrasts involving placeholder positions (*a* vs. *b*, *ab* vs. *c*, *b* vs. *c*, and *bc* vs. *a*) are in Appendix L. For *b* and *c* combined compared to *a*, significant differences across grades exist with $p < .0006$. The only contrast which was not significant at the .05 or lower level was *b* compared to *c*. This was not significant at the .05 level since $p = .0792$. For the open sentences with whole number solutions, the mean correct for placeholder position *a* was 67.17 percent, for *b* was 92.74 percent, and for *c* was 92.78 percent. This seems to clearly indicate students' performance level was the lowest for placeholder *a*. For the open sentences with no whole number solutions, the mean correct for placeholder *a* was 86.38 percent, for *b* was 78.19 percent, and for *c* was 85.49 percent. For the open sentences with no whole number solutions, the students performance level was the lowest for placeholder *b*. If the two means for each placeholder are combined, the following overall means result: a = 76.78 percent, b = 85.46 percent, and c = 89.14 percent. Overall, performance level was lowest for placeholder a.

These results are in agreement with the results from Weaver's (1971) study. In his study, the performance level was consistently lowest for placeholder a. The performance level was highest for placeholder c.

These results are also in agreement with Grouws' (1971) study. Of the four open sentence types he studied, the student performance level was the lowest on the two open sentences which had the placeholder in the a position. These results are also in agreement with Suppes' (1972) results. Open sentences with the placeholder in a position received the lowest percentages of correct responses.

An examination of the third- through sixth-grade textbooks indicates 785 multiplication open sentences had the placeholder in the a position. Only 103 division open sentences however had the placeholder in the a position. The student performance level was the lowest for division open sentences with the placeholder in a position.

When students were presented with either operation-left or operation-right division open sentences with the placeholder in a position, two response patterns clearly predominate. First, if one number of the open sentence does not divide the second number evenly, a high percentage of students respond with "N"--indicating no whole number solution exists. If one number will divide the second number evenly, a high percentage of the students respond with the quotient of the 2 numbers (i.e.,

$2 = \blacksquare \div 7$, 59 percent responded "N," $6 = \blacksquare \div 2$, only $8\frac{3}{4}$ percent responded "N" while 50 percent responded "3"). It appears that in division open sentences, the student sees two numbers, an operation sign, and responds with the quotient (if a whole number quotient exists) of those two numbers. It appears as if the students either disregard the equality sign or mentally turn the open sentence around for their convenience.

It would seem that if the teachers want students to be able to solve open sentences with the placeholders in positions a, b, and c equally well, students will have to be provided with more experiences with division open sentences with the placeholder in the a position.

5. Significant interactions existed among the following factors: grade level, operation, symmetric property, and placeholder position.

An examination of the 11 interaction contrasts indicates seven contrasts were significant at the $p < .01$ level. One contrast was significant at the $p < .05$ level. Three contrasts were not significant at the .05 level.

The two-way interaction contrasts seem to have the highest significance levels. The three- and four-way interactions possibly start canceling each other, therefore not resulting in significant values. Opportunity to learn might account for some of the interactions. As the grade level increases, students have had more experiences with some of the open sentence types. One would therefore expect that as the grade level increases, the performance level on the various factors would increase. One would expect significant interactions between grade and the other factors.

For instance, a significant interaction exists between grade and multiplication and division. In the fourth grade, students have had little experience with division, while having had a year or two of experience with multiplication. By the sixth grade, students have studied multiplication and division for two additional years. One would expect, therefore, that as grade level increased, there would be a change in performance level on multiplication and division open sentences. Similarly, the following interactions exist, Grade x LR, Grade x Placeholder, MD x LR, MD x LR x G, LR x P, and MD x LR x P.

6. The performance level of subjects on open number sentences which have no whole number solutions was significantly different from the performance level of subjects on open sentences which have whole number solutions.

Within each grade level, the mean correct responses for the eight open sentence types having whole number solutions was greater than the mean correct responses for the eight open sentence types having no whole number solutions. The Wilcoxen test indicatd that with $p < .05$, significant differences existed between the mean correct responses offered to the open sentences having whole number solutions and the mean correct responses to the open sentences having no whole number solutions. This seems to indicate that student performance level is higher on open sentences with whole number solutions than on open sentences with no whole number solutions.

These results are in agreement with the results from Weaver's (1971) study. Within grade one, percentages of correct responses differed by 1 percent, being 1 percent higher for the open sentences with no whole number solutions. Within grades two and three however, the performance level was noticeably higher on the open sentences with whole number solutions than on the open sentences with no whole number solutions (grade two, 64% vs. 48%, and grade three, 77% vs. 51%).

A partial explanation for the students higher performance level on open sentences with whole number solutions might be their opportunity to learn. There were no examples of open sentences with no whole number solutions within their textbooks. With one exception in the fourth grade, the open sentence types with no whole number solutions were rated the lowest by the teachers. In other words, teachers recognized that students had not had much exposure to these open sentence types and therefore anticipated their performance level would be low.

It would seem that, if teachers want students to be able to recognize open sentences which have no whole number solutions, experiences with these open sentences will have to be incorporated within the mathematics program.

7. Relative to the open sentences with no whole number solutions, there was no significant difference between students' performance level on multiplication (division) open sentences in which the product (dividend) was a product of the basic fact, and students' performance level on multiplication (division) open sentences in which the "product" (dividend) was not a "product" of some basic fact.

The mean correct responses to the "not a basic fact" open sentences was 80.12 percent, while the mean correct responses to the "basic fact" open sentences was 82.88 percent. The data were tested by the Wilcoxen signed ranks test. No significant differences resulted ($p < .4168$). It appears that students can recognize both types of open sentences (no whole number solution open sentences whose product is a basic fact product and sentences whose "product" is not a basic fact product) equally well.

Recommendations for Future Research

The results reported in this thesis need to be examined for validity with students of different mathematical backgrounds. The present study needs to be extended beyond the set of whole numbers. For example, would similar results occur if the domain was extended to include integers and rational numbers? In terms of instruction, this is extremely important. In the elementary school, most of the student's work is with the set of whole numbers. Will the student's ability to solve problems within the whole number domain be systematically carried over to the other domains? Or, do students need to be guided and/or instructed in order to achieve a systematic transfer of knowledge to the larger number domains?

A study is needed to explore why the results reported in this investigation occurred. Placeholder position a division open sentences were answered incorrectly more often than any other open sentence type. $\blacksquare \div 7 = 4$, $\blacksquare \div 2 = 7$, and $\blacksquare \div 9 = 8$ are examples of operation-left open sentences which were answered correctly only 37 percent to 44 percent of the time. 8 = ■ + 5, 8 = ■ + 6, and 5 = ■ + 3 are examples of operation-right open sentences which were answered correctly only 27 percent to 31 percent of the time. Overall, placeholder position a received the lowest percentage of correct responses. Is there something that makes this placeholder position more difficult? If the students had more experiences with placeholder position a, would the same problems persist?

Multiplication open sentences were answered correctly more frequently than division open sentences. Is this attributable to more experience with one operation than with the other? Is there something intrinsically more difficult about the division operation? In that event, can one expect performance on division open sentences will always consistently lag behind performance on multiplication open sentences?

Operation-left open sentences were answered correctly more frequently than operation-right open sentences. Similar questions should be explored to find out why students' performance level is higher on operation-left open sentences. Given more experiences with operation-right, would the differences in performance levels decrease?

Two major questions are reoccurring. First, is student opportunity to learn the major factor accounting for the low performance level on selected open sentence types? In other words, if students regularly explored and solved all the open sentence types, would the performance levels be approximately equal? Secondly, is there some intrinsic difficulty within some of these open sentence types which makes them more difficult to solve than others?

A study involving the "effectiveness" of teaching a systematic method of solving various open sentence types to students would be of interest also. For example, would students' performance level improve if students were taught the factor product relationship? Given two factors, the students are taught to multiply. Given a factor and a product, the students are instructed to divide in order to find the remaining factor.

A second method of instruction might be the "doing, undoing" commutativity idea. Students could be taught that a product results from multiplying two factors. Therefore, if a product and a factor are given, one can "undo" the product by doing the opposite operation, that is, dividing. If the students learn the "doing, undoing" idea, can they relate one open sentence type to another successfully?

Students could be taught independent methods for solving each open sentence type. Since there are 12 open sentence types, students could be taught 12 rules, one for solving each open sentence.

Many unanswered questions remain concerning open multiplication and division sentences. Answers will come as studies investigating the above questions are conducted. In the mean time, teachers should be made aware of several things. Placeholder position a needs special consideration. One can no longer assume that if students can solve open sentences with the placeholder in position b or c, they will also be able to solve similar open sentences with the placeholder in a position. If teachers expect students to correctly solve placeholder position a open sentences, specific experiences with placeholder position a must be incorporated in the mathematics program. Because students can solve operation-left open sentences is no guarantee they can solve operation-right open sentences. Students need experiences with both types of open sentences. Students give correct responses to division open sentences less frequently than they do to multiplication open sentences. More attention and concern need to be given to division open sentences.

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